RB IIT Academy MAINS PART TEST 4

1) Equation of the plane containing the lines $\bar{r} = \left(\bar{i} - 2\bar{j} + \bar{k}\right) + t\left(\bar{i} + 2\bar{j} - \bar{k}\right)$

$\bar{r} = \left(\bar{i} + 2\bar{j} - \bar{k}\right) + \bar{s}\left(\bar{i} + \bar{j} + 3\bar{k}\right)$ is

A) $\bar{r}.\left(7\bar{i} - 4\bar{j} - \bar{k}\right) = 14$ B) $\bar{r}.\left(\bar{i} + 2\bar{j} - \bar{k}\right) = 10$ C) $\bar{r}.\left(\bar{i} + \bar{j} + 3\bar{k}\right) = 20$

D) $\bar{r}.\left(\bar{i} - 2\bar{j} + \bar{k}\right) = 27$

Correct Answer: A

Solution: Equation of a plane containing the line $\bar{r} = \bar{a} + t\bar{b}$, $\bar{r} = \bar{c} + t\bar{d}$ is $\left[\bar{r} - \bar{a}\ \bar{b}\ \bar{d}\right] = 0$

$\bar{r}.\ \bar{b} \times \bar{d} = \bar{a}.\ \bar{b} \times \bar{d}$

$\bar{b} \times \bar{d} = \left(\bar{i} + 2\bar{j} - \bar{k}\right) \times \left(\bar{i} + \bar{j} + 2\bar{k}\right) = 7\bar{i} - 4\bar{j} - \bar{k}$

$\bar{a}.\ \bar{b} \times \bar{d} = 14.$

Hence equation of required plane is $\bar{r}.\ 7\bar{i} - 4\bar{j} - \bar{k} = 14$

2) If z_1, z_2 and z_3 are the vertices of an isosceles right angled triangle , right angled at the vertex z_2, then $(z_3 - z_2)^2 + (z_1 - z_2)^2 =$

A) 0 B) $(z_1 - z_3)^2$ C) $\left(\frac{z_1 + z_3}{2}\right)^2$ D) None of these

Correct Answer: A

Solution:

we know that $\frac{z_3 - z_2}{z_1 - z_2} = \frac{|z_3 - z_2|}{|z_1 - z_2|}.\ e^{-\frac{i\pi}{2}} \Rightarrow z_3 - z_2 = -i\,(z_1 - z_2) \Rightarrow (z_3 - z_2)^2 + (z_1 - z_2)^2 = 0$

3) If z_1, z_2, z_3 are non-zero complex numbers representing the points A, B, C such that $\frac{2}{z_1} = \frac{1}{z_2} + \frac{1}{z_3}$. Then

A) A, B, C are collinear. B) Circle Passes through points A, B, C has centre at origin O

C) Circle Passes through A, B, C passes through origin D) None of these.

Correct Answer: C

Solution: $\frac{2}{z_1} = \frac{1}{z_2} + \frac{1}{z_3} \Rightarrow \arg\left(\frac{z_2 - z_1}{z_3 - z_1}\right) = \arg\left(-\frac{z_2}{z_3}\right) = \arg\left(\frac{z_2}{z_3}\right) \pm \pi$

$\Rightarrow \arg\left(\frac{z_2 - z_1}{z_3 - z_1}\right) = \arg\left(\frac{z_2 - 0}{z_3 - 0}\right) \pm \pi \Rightarrow \arg\left(\frac{z_2 - z_1}{z_3 - z_1}\right) - \arg\left(\frac{z_2 - 0}{z_3 - 0}\right) = \pm\pi$

Sum of angles at A and origin is$\pm\pi$ Hence points O, B, A, C are concyclic.

4) The complex number 3 + 4i is rotated about origin by an angle of $\frac{\pi}{4}$ and then stretched 2- times. The complex number corresponding to new position is

A) $\sqrt{2}\left(-3+4i\right)$ B) $\sqrt{2}\left(-1+7i\right)$ C) $\sqrt{2}\left(3-4i\right)$ D) $\sqrt{2}\left(-1-7i\right)$

Correct Answer: B
Solution: The new complex number $3+4i$ is rotated by $\frac{\pi}{4}$ then new complex number is
$(3+4i)\,e^{i\pi/4}=(-1+7i)$
is $2\,(3+4i)\,e^{i\pi/4}=\sqrt{2}\,(-1+7i)$

5) If $(a+ib)^5=\alpha+i\beta$ then $(b+ia)^5$ is equal to

A) $\beta-i\alpha$ B) $\beta+i\alpha$ C) $\alpha-\beta$ D) $-\alpha-i\beta$

Correct Answer: B
Solution: $(a+ib)^5=\alpha+i\beta$
Taking complex conjugate
$(a-ib)^5=\alpha-i\beta$
$\left(-i^2a-ib\right)^5=\alpha-i\beta$
$(-i)^5(b+ai)^5=\alpha-i\beta$
$(b+ai)^5=-\frac{\alpha}{i}+\beta$
$=\alpha i+\beta$

6) If $\Delta ABC, sinAsinBsinC=p$, $\cos A\cos B\cos C=q$ then the cubic equation having roots $\tan A,\tan B,\tan C$ is

A) $qx^3-px^2-qx-p=0$ B) $qx^3-px^2+qx-p=0$
C) $qx^3-px^2-(q+1)\,x-p=0$ D) $qx^3-px^2+(q+1)\,x-p=0$

Correct Answer: D
Solution: $\cos(A+B+C)=\cos A\cos B\cos C-\Sigma\sin A\sin B\cos C\Rightarrow\Sigma\tan A\tan B=\frac{1+q}{q}$
Also $\tan A+\tan B+\tan C=\tan A\tan B\tan C=\frac{p}{q}$

7) The reflection of the point $P(1,0,0)$ in the line $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ is

A) (3, -4, -2) B) (5, -8, -4) C) (1, -1, -10) D) (2, -3, 8)

Correct Answer: B

Solution: Let $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8} = r$

Coordinates of any point Q on the given line $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ is $(2r+1, -3r-1, 8r-10)$ for some $r \in R$

So the direction ratios of PQ are $(2r, -3r-1, 8r-10)$

Now PQ is perpendicular to the given line

if $2(2r)-3(-3r-1)+8(8r-10)=0$

$\Rightarrow 77r - 77 = 0 \Rightarrow r = 1$ and the coordinates of Q, the foot of the perpendicular from P on the line are $(3, -4, -2)$.

Let $R(a, b, c)$ be the reflection of P in the given lines when Q is the mid-point of PR

$\Rightarrow \frac{a+1}{2} = 3, \ \frac{b}{2} = -4, \ \frac{c}{2} = -2$

$\Rightarrow a = 5, b = -8, c = -4$

$\Rightarrow$ Coordinates of the required point are $(5, \ -8, \ -4)$.

8) The ratio in which the plane $\overline{r}.\left(\hat{i}-2\hat{j}+3\hat{k}\right)=17$ divides the line joining the points $-2\hat{i}+4\hat{j}+7\hat{k}$ and $3\hat{i}-5\hat{j}+8\hat{k}$ is

A) 1:5 B) 1:10 C) 3:5 D) 3:10

Correct Answer: D

Solution: Plane is $\overline{r}.\left(\bar{i}-2\bar{j}+3\overline{k}\right)=17 \ldots\ldots\ldots (i)$

A point P dividing the join of $-2\bar{i}+4\bar{j}+7\overline{k}$ and $3\bar{i}-5\bar{j}+8\overline{k}$ on the ratio $\lambda : 1$ is $\frac{3\lambda-2}{\lambda+1}\bar{i}+\frac{-5\lambda+4}{\lambda+1}\bar{j}+\frac{8\lambda+7}{\lambda+1}\overline{k}$

it lies (1) then we get $\lambda = \frac{3}{10} \Rightarrow \lambda : 1 = 3 : 10$

(or)

Ratio of Perpendicular distances from the points $(-2, 4, 7)$ and $(3, -5, 8)$ to the plane $x - 2y + 3z = 17$ is $6 : 10 = 3 : 5$

9) Let $f(x) = 2x^3 + ax^2 + bx$ where $a, b \in N$ if graph of $f(x)$ cuts the x–axis at 3 distinct points, then minimum value of $a^2 + b^2$ is

A) 10 B) 2 C) 9 D) 4

Correct Answer: A

Solution:

$f(x) = 2x^3 + ax^2 + bx$

$\Rightarrow f'(x) = 6x^2 + 2ax + b.$

If $f(x) = 0$ has 3 distinct real roots, then $f'(x) = 0$ must have 2 distinct real roots for it $4a^2 - 24b > 0 \Rightarrow a^2 > 6b$

$\Rightarrow a \geq 3, b \geq 1$.

Minimum value of $a^2 + b^2$ will be obtained at $a = 3, b = 1$

$a^2 + b^2 = 9 + 1 = 10$

10) If $p_1 p = 2\left(q_1 + q\right)$ then which of following statement about the quadratic equations $x^2 + px + q = 0;\ x^2 + p_1 x + q_1 = 0$ is always true Here $p, p_1, q, q_1, \in R$

A) Both the equation ha real roots B) Both the equations have imaginary roots

C) At least one of the equations has real roots D) Exactly one of them has equal roots

Correct Answer: C

Solution: $\Delta_1 = p^2 - 4q; \Delta_2 = p_1^2 - 4q_1$

$\Delta_1 + \Delta_2 = p^2 + p_1^2 - 4\left(q + q_1\right) = p^2 + p_1^2 - 2p_1 p = \left(p - p_1\right)^2 \geq 0$

Hence both Δ_1, Δ_2 can of be negative.

That is at least one of the equation has real roots.

11) If the equation $x^2 + 9y^2 - 4x + 3 = 0$ is satisfied for real values of x and y then

A) $x \in [1, 3], y \in [1, 3]$ B) $x \in [1, 3], y \in \left[\frac{-1}{3}, \frac{1}{3}\right]$ C) $x \in \left[\frac{-1}{3}, \frac{1}{3}\right], y \in [1, 3]$

D) $x \in \left[\frac{-1}{3}, \frac{1}{3}\right], y \in \left[\frac{-1}{3}, \frac{1}{3}\right]$

Correct Answer: B

Solution: Given equation $x^2 + 9y^2 - 4x + 3 = 0$

or $x^2 - 4x + 9y^2 + 3 = 0.$

Since x is real $(-4)^2 - 4\left(9y^2 + 3\right) \geq 0$

$\Rightarrow 16 - 4\left(9y^2 + 3\right) \geq 0$

$\Rightarrow 4 - 9y^2 - 3 \geq 0$

$\Rightarrow 9y^2 - 1 \leq 0$

$y^2 \leq \frac{1}{9} \Leftrightarrow -\frac{1}{3} \leq y \leq \frac{1}{3}$

Equation (i) can also be written as $9y^2 + 0y + x^2 - 4x + 3 = 0$

Since y is real $0^2 - 4.9\left(x^2 - 4x + 3\right) \geq 0$

$\Rightarrow x \in [1, 3]$

12) For a non zero polynomial P, the equation $|P(x)| = e^x$ has

A) At least one solution B) No solution C) Exactly 2 solution D) Exactly 1 solution

Correct Answer: A

Solution: To check number of solutions of the equation $e^{-x} = |P(x)|$ it is enough to check the number of the solutions of the equation $e^{-x} \times |P(x)| = 1$

$\underset{x\to\infty}{Lim}\, e^{-x}|P(x)| = 0$ and $\underset{x\to-\infty}{Lt}\, e^{-x}|P(x)| = \infty$.

Consequently there is an $x_0 \in R$ such that $e^{-x_0}|P(x_0)| = 1$

Since product of polynomial and exponential functions are continious and its range $(0, \infty) \Rightarrow$ there is atleast one x_0 for which function accepts number 1 aand which is the solution of the given equation.

13) If $ax^2 + bx + c = 0$ has no real zeroes, and if $c < 0$, then

A) a< 0 B) a + b + c > 0 C) 4a + 2b + c > 0 D) a – b + c > 0

Correct Answer: A

Solution: Let $f(x) = ax^2 + bx + c$, Since $f(x) = 0$ has no real zeroes, either $f(x) > 0$ or $f(x) < 0$ for all.

since $f(0) = c < 0$ we get $f(x) < 0$ for all $x \in R$.

Therefore, $a < 0$ as the parabola $y = f(x)$ must open downward. Obviously $f(1), f(-1)$ and $f(2) < 0$.

14) The equation $|2ax - 3| + |ax + 1| + |5 - ax| = \frac{1}{2}$ possesses

A) Infinite number of real solution for some $a \in R$

B) Finite number of real solutions for some $a \in R$ C) No real solution for some $a \in R$

D) No real solution for all $a \in R$

Correct Answer: D

Solution: The expression

$|2ax - 3| + |ax + 1| + |5 - ax| \geq |2ax - 3 + (-ax - 1) + 5 - ax| = 1$

Based on in-equality on modulus.

Hence given equation has no solution.

15) A, B, C are the points on x, y and z axes respectively in a three dimensional co-ordinate system with O as origin. Suppose the area of triangles OAB,OBC and OCA are 4, 12 and 6 respectively, then the area of the triangle ABC equals

A) 16 B) 14 C) 28 D) 32

Correct Answer: B

Solution: Area of triangle ABC = $\frac{1}{2}\overline{AB} \times \overline{AC} = \frac{1}{2}\bar{a} \times \bar{b} + \frac{1}{2}\bar{b} \times \bar{c} + \frac{1}{2}\bar{c} \times \bar{a}$

$= \frac{1}{2}\Delta ABO + \frac{1}{2}\Delta CBO + \frac{1}{2}\Delta ACO$

$[ABC] = \sqrt{[OAB]^2 + [OBC]^2 + [OCA]^2}$

where $[ABC]$ = $\sqrt{4^2 + 12^2 + 6^2} = \sqrt{196} = 14$

16) Equation of the plane passing through the origin and perpendicular to the planes $x + 2y + z = 1, 3x - 4y + z = 5$

A) $x + 2y - 5z = 0$ B) $x - 2y - 3z = 0$ C) $x - 2y + 5z = 0$ D) $3x + y - 5z = 0$

Correct Answer: D

Solution: Let $ax + by + cz + d = 0$ be the plane perependicular to the given planes

Given plane is passing through origin $\Rightarrow d = 0$

$x + 2y + z - 1 = 0$ and $3x - 4y - z - 5 = 0$

$a + 2b + c = 0$ and $3a - 4b - c = 0, a, b$ and c are obtained by taking the cross product of vectors $i + 2j + k, 3i - 4j + k = 6i + 2j - 5k$

Equation of required plane is $6x + 2y - 10z = 0 \Rightarrow 3x + y - 5z = 0$

17) $\bar{a}, \bar{b}, \bar{c}$ are three unit vectors equally inclined to each other at an angle $\frac{\pi}{3}$ then the value of

$\left[\bar{a} + \bar{b}\ \ \bar{b} + \bar{c}\ \ \bar{c} + \bar{a}\right]^2 + \left[\bar{a} \times \bar{b}\ \ \bar{b} \times \bar{c}\ \ \bar{c} \times \bar{a}\right]$ is

A) $\frac{3}{4}$ B) $\frac{5}{2}$ C) $\frac{7}{2}$ D) 4

Correct Answer: B

Solution: $\left[\bar{a}\ \bar{b}\ \bar{c}\right]^2 = \begin{vmatrix} a.a & a.b & a.c \\ b.a & b.b & b.c \\ c.a & c.b & c.c \end{vmatrix} = \begin{vmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 1 \end{vmatrix} = \frac{1}{2}$

$\left[\bar{a} + \bar{b}\ \ \bar{b} + \bar{c}\ \ \bar{c} + \bar{a}\right]^2 + \left[\bar{a} \times \bar{b}\ \ \bar{b} \times \bar{c}\ \ \bar{c} \times \bar{a}\right] = \left(2\left[\bar{a}\,\bar{b}\,\bar{c}\right]\right)^2 + \left[\bar{a}\,\bar{b}\,\bar{c}\right]^2 = 5\left[\bar{a}\,\bar{b}\,\bar{c}\right]^2 = \frac{5}{2}$

18) If $|\bar{a}| = 3, |\bar{b}| = 4$ and $|\bar{c}| = 5$ then $|\bar{a} - \bar{b}|^2 + |\bar{b} - \bar{c}|^2 + |\bar{c} - \bar{a}|^2$ does not exceed

A) 25 B) 50 C) 75 D) 150

Correct Answer: D

Solution: $\left|\bar{a} + \bar{b} + \bar{c}\right| = 0$

Squaring both sides we get

$|\bar{a}|^2 + |\bar{b}|^2 + |\bar{c}|^2 + 2(\bar{a}.\bar{b} + \bar{b}.\bar{c} + \bar{c}.\bar{a}) = 0$

$\bar{a}.\bar{b}+\bar{b}.\bar{c}+\bar{c}\bar{a}\geq -25$

$\left|\bar{a}-\bar{b}\right|^2+|b-\bar{c}|^2+|\bar{c}-\bar{a}|^2$

$=2(|\bar{a}|^2+|\bar{b}|^2+|\bar{c}|^2-\bar{a}.\bar{b}-\bar{b}.\bar{c}-\bar{c}.\bar{a})$

$\leq 2(50+25)=150$

19) Angle between the vectors $\vec{a}$ and $\vec{b}$,w here $\vec{a}$, $\vec{b}$, $\vec{c}$are unit vectors satisfying $\vec{a}+\vec{b}+\sqrt{3}\vec{c}=\vec{0}$ is

A) $\frac{\pi}{2}$ B) $\frac{\pi}{6}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{3}$

Correct Answer: D

Solution: $\vec{a}+\vec{b}=-\sqrt{3}\vec{c}$

$|\vec{a}|^2+|\vec{b}|^2+2\vec{a}.\vec{b}=3|\vec{c}|^2$

$\Rightarrow 2+2|\vec{a}||\vec{b}|\cos\theta=3$

$\Rightarrow \cos\theta=\frac{1}{2}\Rightarrow\theta=\frac{\pi}{3}$

20) The value of $Cot16^0cot44^0+cot44^0cot76^0-cot76^0cot16^0$ is

A) 3 B) $\frac{1}{3}$ C) $-\frac{1}{3}$ D) -3

Correct Answer: A

Solution: $Cot16^\circ Cot44^\circ+Cot44^\circ Cot76^\circ-Cot76^\circ Cot16^\circ$

Adding and subtracting 3 we get

$=(Cot16^\circ Cot44^\circ-1)+(Cot44^\circ Cot76^\circ-1)-(Cot76^\circ Cot16^\circ+1)+3$

$Cot(A+B)=\frac{CotACotB-1}{CotA+CotB}$

$\Rightarrow CotACotB-1=Cot(A+B)(CotA+CotB)$

$Cot(16^o+44^o)(Cot16^o+Cot44^o)=Cot16^oCot44^o-1$

$\Rightarrow Cot16^oCot44^o-1=\sqrt{3}(Cot16^o+Cot44^o)$

$Cot(44^o+76^o)(Cot76^o+Cot44^o)=Cot76^oCot44^o-1$

$\Rightarrow Cot76^oCot44^o-1=-\sqrt{3}(Cot76^o+Cot44^o)$

$Cot(A-B)=\frac{CotACotB+1}{CotB-CotA}$

$\Rightarrow CotACotB+1=Cot(A-B)(CotB-CotA)$

$\Rightarrow Cot76^oCot16^o+1=Cot(76^o-16^o)(Cot16^o-Cot76^o)$

$=-\sqrt{3}(Cot76^o-Cot16^o)$

$=(Cot16^\circ Cot44^\circ-1)+(Cot44^\circ Cot76^\circ-1)-(Cot76^\circ Cot16^\circ+1)+3$

$\sqrt{3}(Cot16^o+Cot44^o)-\sqrt{3}(Cot76^o+Cot44^o)+\sqrt{3}(Cot76^o-Cot16^o)+3$

$=0+3=3$

21) If in ΔABC, $\tan A + \tan B + \tan C > 0$ then the triangle is

A) acute angled B) obtuse angled C) right angled D) nothing can be said

Correct Answer: A

Solution: Given $\tan A + \tan B + \tan C > 0$

If $A + B + C = \pi$ then $TanA + TanB + TanC = TanA.TanB.TanC$

$\tan A + \tan B + \tan C > 0 \Rightarrow TanATanBTanC > 0$

If two of tan functions are negative another is positive we get two obtuse angles , which is not possible.

If one of the angle is negative two ar positive product cannot be positive

Hence each tan function must be positive.

$\Rightarrow A, B, C$ are all acute angles

22) The minimum value of $(Sinx + Cosecx)^2 + (Cosx + Secx)^2 \ \ \forall x \in R$ is

A) 7 B) 8 C) 9 D) 10

Correct Answer: C

Solution: Let $y = (Sinx + Cosecx)^2 + (Cosx + Secx)^2$

$y = (Sin^2x + Cos^2x) + 2(SinxCosecx + CosxSecx) + Sec^2x + Cosec^2x$

$1 + 2(1 + 1) + 1 + Tan^2x + 1 + Cot^2x$

$= 7 + (Tanx - Cotx)^2 + 2TanxCotx$

$= 7 + (Tanx - Cotx)^2 + 2$

$= 9 + (Tanx - Cotx)^2 \geq 9$

23) If $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ and $\sum_{n=1}^{\infty} \frac{1}{\tan^n\theta} = \sin\theta + \cos\theta$ then the value of $\tan\theta$ is

A) $\sqrt{3}$ B) $\sqrt{2} + 1$ C) $2 + \sqrt{3}$ D) $\sqrt{2}$

Correct Answer: A

Solution: $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right) \Rightarrow \tan\theta > 1 \Rightarrow 0 < \frac{1}{\tan\theta} < 1$

$\sum_{n=1}^{\infty} \frac{1}{\tan^n\theta} = \frac{\left(\frac{1}{\tan\theta}\right)}{1 - \frac{1}{\tan\theta}} = \frac{1}{\tan\theta - 1} = \frac{\cos\theta}{\sin\theta - \cos\theta}$

$\sum_{n=1}^{\infty} \frac{1}{\tan^n\theta} = \sin\theta + \cos\theta \Rightarrow \frac{\cos\theta}{\sin\theta - \cos\theta} = \sin\theta + \cos\theta$

$\cos\theta = \sin^2\theta - \cos^2\theta$

$2\cos^2\theta + \cos\theta - 1 = 0$

$\cos\theta = \frac{1}{2} \ . \ \Rightarrow \tan\theta = \sqrt{3}$

24) The value of $^{12}C_2 + {}^{13}C_3 + {}^{14}C_4 + \ldots\ldots + {}^{999}C_{989}$

A) $^{1000}C_{11} - 12$ B) $^{1000}C_{11} + 12$ C) $^{999}C_{11} - 12$ D) $^{1000}C_{989}$

Correct Answer: A
Solution: Adding and subtracting $^{12}C_1$ to the given expression
$^{12}C_2 + {}^{13}C_3 + {}^{14}C_4 + \ldots\ldots + {}^{999}C_{989}$
we ge $^{12}C_1 + {}^{12}C_2 + {}^{13}C_3 + {}^{14}C_4 + \ldots\ldots + {}^{999}C_{989} - {}^{12}C_1$
By using formula $^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$
for pairs of terms we get $^{1000}C_{989}$
For example $^{12}C_1 + {}^{12}C_2 = {}^{13}C_2$
Hence value of final expression = $^{1000}C_{989} - 12 = {}^{1000}C_{11} - 12$

25) The Coefficient of x^9 in $\left(x - {}^{21}C_0\right)\left(x - {}^{21}C_1\right)\left(x - {}^{21}C_2\right)\ldots\ldots\ldots\left(x - {}^{21}C_{10}\right) is$

A) $2^{40} - \frac{1}{2}\,{}^{42}C_{20}$ B) $2^{39} - \frac{1}{2}\,{}^{42}C_{21}$ C) $2^{40} - {}^{42}C_{20}$ D) $2^{39} - \frac{1}{4}\,{}^{42}C_{21}$

Correct Answer: D
Solution: Coefficient of x^9 = Sum of products of $^{21}C_0, {}^{21}C_1 \ldots\ldots\ldots\ldots {}^{21}C_{10}$. taking '2' at a time, we get

$\sum_{j=0}^{10}\sum_{r=0}^{10} {}^{21}C_r\,{}^{21}C_j$ where $r < j$.

$$\sum_{j=0}^{10}\sum_{r=0}^{10} {}^{21}C_r\,{}^{21}C_j = \frac{\sum_{j=0}^{20}\sum_{r=0}^{20} {}^{21}C_r\,{}^{21}C_j}{2}$$

$$(C_0 + C_1 + \ldots\ldots C_{20} + C_{21})^2 = {C_0}^2 + {C_1}^2 + \ldots\ldots {C_{21}}^2 + 2\sum_{j=0}^{21}\sum_{r=0}^{21} {}^{21}C_r\,{}^{21}C_j where\, r \neq j$$

$\Rightarrow 2^{42} = {}^{42}C_{21} + 2X$ where

$$X = \sum_{j=0}^{20}\sum_{r=0}^{20} {}^{21}C_r\,{}^{21}C_j where\, r \neq j$$

$X = 2^{41} - {}^{41}C_{20}$

26) If $A = \begin{bmatrix} 0 & \alpha \\ 0 & 0 \end{bmatrix}$ and $(A+I)^{50} - 50A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ .Then value of $a + b + c + d =$

A) 0 B) 1 C) 2 D) 4

Correct Answer: C
Solution: $(A+I)^{50} = \begin{bmatrix} 0 & 50\alpha \\ 0 & 0 \end{bmatrix}$

$(A+I)^{50}-50A=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$a+b+c+d=2$

27) Equations $3x-2y+z=0,\ \lambda x-14y+15z=0,\ \ x+2y-3z=0$ have solutions other than $x=y=z=0$ then $\lambda=$

A) 1 B) 2 C) 3 D) 5

Correct Answer: D

Solution: If homogenious equation has non-singular solution then $\begin{vmatrix}3 & -2 & 1\\ \lambda & -14 & 15\\ 1 & 2 & -3\end{vmatrix}=0\Rightarrow\lambda=5$

28) If 7 divide $32^{32^{32}}$ then remainder is

A) 1 B) 0 C) 4 D) 6

Correct Answer: C

Solution: $32=2^5\Rightarrow(32)^{32}=\left(2^5\right)^{32}$

$=2^{160}=(3-1)^{160}=3m+1, m\in N$

$(32)^{32^{32}}=(32)^{3m+1}=2^{5(3m+1)}$

$2^{3(5m+1)}\ 2^2=4.8^{5m+1}$

$4(7+1)^{5m+1}=4\left(7n+1\right), n\in N=28n+4$

∴ Remiander = 4

29) If the number of terms in $\left(1+x^{-1}+x^{-2}\right)^n$ is 53 then the larest prime p so that n! is divisible by 5^{p-1}

A) 3 B) 5 C) 7 D) 11

Correct Answer: C

Solution: $2n+1=53n=26$

so exponent of 5 in $(26)!$ is = $\left[\frac{26}{5}\right]+\left[\frac{26}{5^2}\right]+\ldots\ldots=6$

Therefore $p=7$

30) If n is even positive integer, then the condition that the greatest term in the expansion $(1+x)^n$ of may have the greatest coefficient also is

A) $\frac{n}{n+2} < x < \frac{n+2}{n}$ B) $\frac{n+1}{n} < x < \frac{n}{n+1}$ C) $\frac{n}{n+4} < x < \frac{n+4}{n}$ D) None of these

Correct Answer: A

Solution: Let $n = 2m$ then the greatest term in $(1+x)^{2m}$ is

$T_{m+1} \Rightarrow \frac{T_{m+1}}{T_m} > 1, \frac{T_{m+1}}{T_{m+2}} > 1 \Rightarrow \frac{m}{m+1} < x < \frac{m+1}{m}$

i.e $\frac{n}{n+2} < x < \frac{n+2}{n}$

31) Imagine a light planet revolving around a very massive star in a circular orbit of radius R with a period of revolution T. If the gravitational force of attraction between planet and star is proportional to $R^{-\frac{5}{2}}$, then T^2 is proportional to

A) R^3 B) $R^{7/2}$ C) $R^{5/2}$ D) $R^{3/2}$

Correct Answer: B

Solution: For revolution of planet centripetal force is provided by gravitational force of attraction

$m\omega^2 R \propto R^{-5/2} \Rightarrow \frac{1}{T^2} \propto R^{-7/2} \Rightarrow T^2 \propto R^{7/2}$

32) A rocket of mass M is launched vertically from the surface of the earth with an initial speed V. Assuming the radius of the earth to be R and negligible air resistance, the maximum height attained by the rocket above the surface of the earth is

A) $R/\left(\frac{gR}{2V^2} - 1\right)$ B) $R\left(\frac{gR}{2V^2} - 1\right)$ C) $R/\left(\frac{2gR}{V^2} - 1\right)$ D) $R\left(\frac{2gR}{V^2} - 1\right)$

Correct Answer: C

Solution: $\Delta K.E. = \Delta U \Rightarrow \frac{1}{2}MV^2 = GM_eM\left(\frac{1}{R} - \frac{1}{R+h}\right) \ldots (i)$

Also $g = \frac{GM_e}{R^2} \ldots . (ii)$

On solving (i) and $(ii) h = \frac{R}{\left(\frac{2gR}{V^2} - 1\right)}$

33) A projectile is projected with velocity kv_e in vertically upward direction from the ground into the space. $(v_e$ is escape velocity and $k < 1)$. If air resistance is considered to be negligible then the maximum height from the centre of earth to which it can go, will be : R = radius of earth

A) $\frac{R}{k^2+1}$ B) $\frac{R}{k^2-1}$ C) $\frac{R}{1-k^2}$ D) $\frac{R}{k+1}$

Correct Answer: C

Solution: Kinetic energy = Potential energy $\frac{1}{2}m\left(kv_e\right)^2 = \frac{mgh}{1+\frac{h}{R}}$

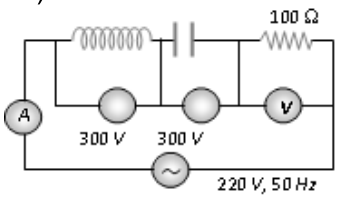
$\Rightarrow \frac{1}{2}mk^2 \times 2gR = \frac{mgh}{1+\frac{h}{R}} \Rightarrow h = \frac{Rk^2}{1-k^2}$

Height of Projectile from the earth's surface = h

Height from the centre $r = R + h = R + \frac{Rk^2}{1-k^2}$

By solving $r = \frac{R}{1-k^2}$

34) In the circuit shown below, what will be the readings of the voltmeter and ammeter



A) $800V, 2.2A$ B) $300V, 2A$ C) $220V, 2.2A$ D) $100V, 2.1A$

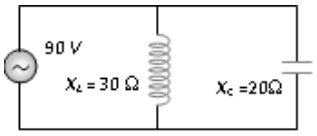
Correct Answer: C

Solution: $V^2 = V_R^2 + (V_L - V_C)^2$

$V_R = V = 220\ V$

Also, $i = \frac{220}{100} = 2.2\ A$

35) In the adjoining figure the impedance of the circuit will be



A) $120\ ohm$ B) $50\ ohm$ C) 60 ohm D) 90 ohm

Correct Answer: C

Solution: $i_L = \frac{90}{30} = 3\ A,$

$i_C = \frac{90}{20} = 4.5\ A$

Net current through circuit$i = i_C - i_L\ = 1.5\ A$

$Z = \frac{V}{i} = \frac{90}{1.5} = 60\Omega$

36) Match the following

current	R.M.S values
1) $x_0 \sin \omega t$	$i) x_0$
2) $x_0 \sin \omega t \cos \omega t$	$ii) \frac{x_0}{\sqrt{2}}$
3) $x_0 \sin \omega t + x_0 \cos \omega t$	$iii) \frac{x_0}{(2\sqrt{2})}$

A) 1. (i), 2. (ii), 3. (iii) B) 1. (ii), 2. (iii), 3. (i) C) 1. (i), 2. (iii), 3 (ii) D) None of these

Correct Answer: C

Solution: $\overline{i^2} = \frac{\int i^2 dt}{\int dt} = \frac{\int_2^4 (4t) dt}{\int_2^4 dt} = \frac{4\int_2^4 t\ dt}{2} = 2\left[\frac{t^2}{2}\right]_2^4 = \left[t^2\right]_2^4 = 12$

$\Rightarrow i_{rms} = \sqrt{\overline{i^2}} = \sqrt{12} = 2\sqrt{3}\ A$

37) Two equal charges q of opposite sign separated by a distance $2a$

constitute an electric dipole of dipole moment p. If P is a point at a distance r from the centre of the dipole and the line joining the centre of the dipole to this point makes an angle θ with the axis of the dipole, then the potential at P is given by $(r >> 2a)$ (Where $p = 2qa$)

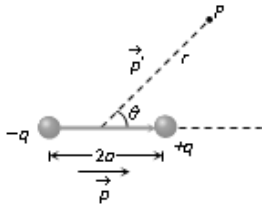
A) $V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$ B) $V = \frac{p\cos\theta}{4\pi\varepsilon_0 r}$ C) $V = \frac{p\sin\theta}{4\pi\varepsilon_0 r}$ D) $V = \frac{p\cos\theta}{2\pi\varepsilon_0 r^2}$

Correct Answer: A

Solution: For the given situation, diagram can be drawn as follows

As shown in figure component of dipole moment along the line OP will be $p' = p\cos\theta$. Hence electric potential at point P will be

$V = \frac{1}{4\pi\varepsilon_0}.\frac{p\cos\theta}{r^2}$



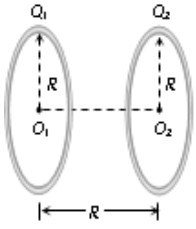
38) Two identical thin rings each of radius R meters are coaxially placed at a distance R meters apart. If Q1 coulomb and Q2 coulomb are respectively the charges uniformly spread on the two rings, the work done in moving a charge qfrom the centre of one ring to that of other is

A) Zero B) $\frac{q(Q_1-Q_2)(\sqrt{2}-1)}{\sqrt{2}.4\pi\varepsilon_0 R}$ C) $\frac{q\sqrt{2}(Q_1+Q_2)}{4\pi\varepsilon_0 R}$ D) $\frac{q(Q_1+Q_2)(\sqrt{2}+1)}{\sqrt{2}.4\pi\varepsilon_0 R}$

Correct Answer: B

Solution: $W = q\left(V_{O_2} - V_{O_1}\right)$

and $V_{O_2} = \frac{Q_2}{4\pi\varepsilon_0 R} + \frac{Q_1}{4\pi\varepsilon_0 R\sqrt{2}}$



$V_{O_2} - V_{O_1} = \frac{(Q_2-Q_1)}{4\pi\varepsilon_0 R}\left[1 - \frac{1}{\sqrt{2}}\right]$

SO, $W = \frac{q.(Q_2-Q_1)}{4\pi\varepsilon_0 R}\frac{(\sqrt{2}-1)}{\sqrt{2}}$

39) A charge $+q$ is fixed at each of the points $x = x_0,\ x = 3x_0,\ x = 5x_0$ ..... infinite, on the $x-$axis and a charge $-q$ is fixed at each of the points $x = 2x_0,\ x = 4x_0, x = 6x_0$,..... infinite. Here x_0 is a positive constant. Take the electric potential at a point due to a charge Q at a distance r from it to be $Q/(4\pi\varepsilon_0 r)$. Then, the potential at the origin due to the above system of charges is

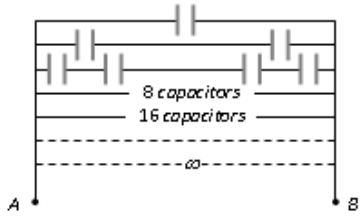
A) 0 B) $\frac{q}{8\pi\varepsilon_0 x_0 \ln 2}$ C) ∞ D) $\frac{q \ln 2}{4\pi\varepsilon_0 x_0}$

Correct Answer: D

Solution: $V = \frac{q}{4\pi\varepsilon_0 x_0}\left[1 + \frac{1}{3} + \frac{1}{5} + \ldots\right] - \frac{q}{4\pi\varepsilon_0 x_0}\left[\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \ldots\right]$

$= \frac{q}{4\pi\varepsilon_0 x_0}\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots.\right] = \frac{q}{4\pi\varepsilon_0 x_0}\log_e 2$

40) An infinite number of identical capacitors each of capacitance $1\mu F$ are connected as in adjoining figure. Then the equivalent capacitance between A and $B$is



A) $1\mu F$ B) $2\mu F$ C) $\frac{1}{2}\mu F$ D) ∞

Correct Answer: B

Solution: This combination forms a G.P.$S = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}\ldots\ldots$

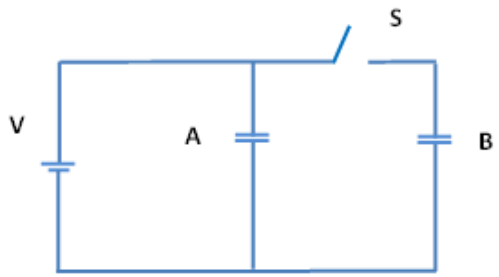
Sum of infinite G.P.$S = \frac{a}{1-r}$

Here a = first term = 1 and r = common ratio $= \frac{1}{2}$

$\Rightarrow S = \frac{1}{1-\frac{1}{2}} = 2$

$\Rightarrow C_{eq} = 2\mu F$

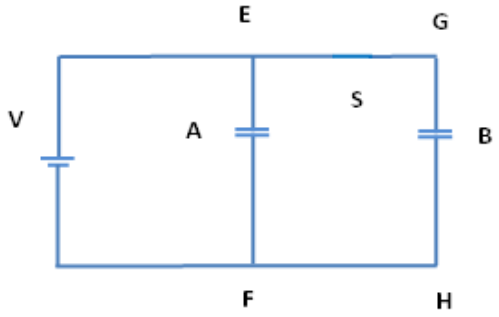
41) Figure given below shows two identical parallel plate capacitors connected to a battery with switch S closed. The switch is now opened and the free space between the plate of capacitors is filled with a dielectric of dielectric constant 3. What will be the ratio of total electrostatic energy stored in both capacitors before and after the introduction of the dielectric



A) 3 B) 5 C) 0.6 D) 1.6

Correct Answer: C

Solution:



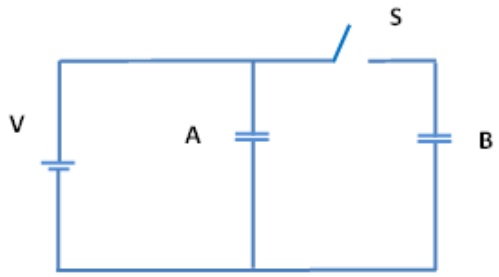
Initially, when the switch is closed, both the capacitors A and B are in parallel.

$\Rightarrow$ potential difference across EF and GH is same.

Hence energy of the system is

$U_1 = \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = CV^2 \ldots . (i)$

In the second case when key K is opened (B) gets disconnected from the battery.



The capacitor B is now isolated, and the charge on an isolated capacitor remains constant, often referred to as bound charge.

On the other hand, A remains connected to the battery. Hence, potential V remains constant on it.

When the capacitors are filled with dieletric, then capacitance of B increases to KC and potential drops to $\frac{V}{K}$ and charge remains same.

On the other hand capacitence of the capacitor increases to K times where as no change in potential difference.

Given : Dilelectric constant of capacitor $k = 3$

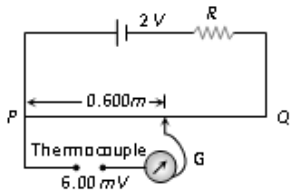
Capacitance of both the capacitors becomes 3C, while potential difference across A is V and potential difference across B is $\frac{V}{3}$ hence energy of the system now is

$U_2 = \frac{1}{2} \times (3C)V^2 + \frac{1}{2} \times (3C) \times (\frac{V}{3})^2$

$U_2 = \frac{10}{6}CV^2 \ldots \ldots . (ii)$

$\therefore \frac{U_1}{U_2} = \frac{3}{5}$

42) Figure shows a simple potentiometer circuit for measuring a small e.m.f. produced by a thermocouple. The meter wire PQ has a resistance $5\Omega$and the driver cell has an e.m.f. of $2V$ If a balance point is obtained0.600 m along $PQ$when measuring an e.m.f. of 6.00 $m.V$ value of resistance $R\Omega$ then R =

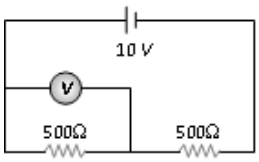


A) 995 B) 1995 C) 2995 D) None of these

Correct Answer: A

Solution: The voltage per unit light of the metre wire PQ is $\left(\frac{6.00\ mV}{0.600\ m}\right)$ i.e. $10\ mV/m$ Hence potential difference across the metre wire is $10\ mV/m \times 1m = 10\ mV$ The current drawn from the driver cell is $i = \frac{10\ mV}{5\ \Omega} = 2\ mA$ The resistance $R = \frac{(2V-10mV)}{2\ mA} = \frac{1990\ mV}{2\ mA} = 995\ \Omega.$

43) A voltmeter of resistance 1000 Ω is connected across a resistance of 500 Ω in the given circuit. What will be the reading of voltmeter



A) $1V$ B) $2V$ C) $6V$ D) $4V$

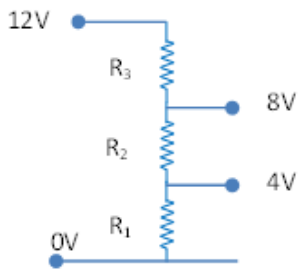
Correct Answer: D

Solution: Total current through the circuit

$$i = \frac{10}{\frac{1000}{3}+500} = \frac{3}{250}A$$

Now voltmeter reading $= i_v \times R_V = \frac{2}{3} \times \frac{3}{250} \times 500 = 4V.$

44) A potential divider is used to give outputs of $4V$ and $8V$ from a $12V$ source. Which combination of resistances, R_1, R_2, R_3 gives the correct voltages i.e $R_1 : R_2 : R_3$



A) 2 : 1 : 2 B) 1 : 1 : 1 C) 2 : 2 : 1 D) 1 : 1 : 2

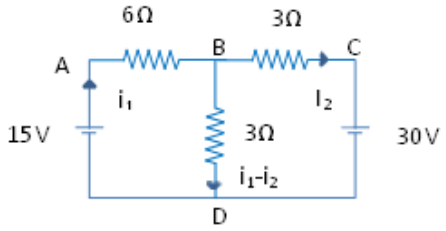
Correct Answer: B

Solution: Resistors are connected in series. So current through each resistor will be same

$\Rightarrow i = \frac{12-8}{R_3} = \frac{8-4}{R_2} = \frac{4-0}{R_1}$

$\Rightarrow \frac{4}{R_3} = \frac{4}{R_2} = \frac{4}{R_1}$ So,$R_1 : R_2 : R_3 = 1 : 1 : 1$

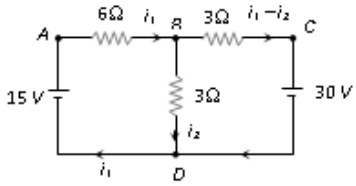
45) In the circuit shown in figure, find the current through the branch BD



A) $5A$ B) $0A$ C) $3A$ D) $4A$

Correct Answer: A

Solution: The current in the circuit are assumed as shown in the fig



Applying KVL along the loop ABDA, we get

$-6i_1 - 3i_2 + 15 = 0\, or\, 2i_1 + i_2 = 5...(i)$

Applying KVL along the loop BCDB, we get

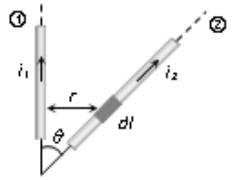
$-3\left(i_1 - i_2\right) - 30 + 3i_2 = 0\ or\ -i_1 + 2i_2 = 10...(ii)$

Solving equation (i) and (ii) for$i_2$we get $i_2\ i_2 = 5A.$

46) Wires 1 and 2 carrying currents i_1 and i_2 respectively are inclined at an angle θ to each other.

What is the force on a small element dl of wire 2 at a distance of r from wire 1 (as shown in figure)

due to the magnetic field of wire1



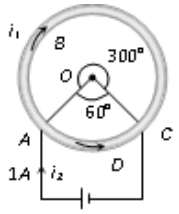
A) $\frac{\mu_0}{2\pi r} i_1\, i_2\, dl\, \tan\theta$ B) $\frac{\mu_0}{2\pi r} i_1\, i_2\, dl\, \sin\theta$ C) $\frac{\mu_0}{2\pi r} i_1\, i_2\, dl\, \cos\theta$ D) $\frac{\mu_0}{4\pi r} i_1\, i_2\, dl\, \sin\theta$

Correct Answer: C

Solution: Length of the component dl which is parallel to wire (1) is $dl\cos\theta$

, so force on it $F = \frac{\mu_0}{4\pi}.\frac{2i_1 i_2}{r}(dl\cos\theta) = \frac{\mu_0 i_1 i_2 dl\cos\theta}{2\pi r}$

47) A cell is connected between the points A and C of a circular conductor ABCD of centre O with angle $AOC = 60^o$. If B_1 and B_2 are the magnitudes of the magnetic fields at O due to the currents in ABC and ADC respectively, the ratio $\frac{B_1}{B_2}$ is



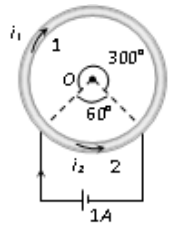
A) 0.2 B) 6 C) 1 D) 5

Correct Answer: C

Solution: $B = \frac{\mu_0}{4\pi}\frac{\theta\, i}{r} \Rightarrow B \propto \theta\, i\; But\; \left(\frac{i_1}{i_2} = \frac{l_2}{l_1} = \frac{\theta_2}{\theta_1}\right)$

$\Rightarrow \frac{B_1}{B_2} = \frac{\theta_1}{\theta_2}.\frac{i_1}{i_2}$

so, $\frac{B_1}{B_2} = \frac{\theta_1}{\theta_2} \times \frac{\theta_2}{\theta_1} \Rightarrow B_1 = B_2$



48) Two particles each of mass m and charge q are attached to the two ends of a light rigid rod of length 2R.The rod is rotated at constant angular speed about a perpendicular axis passing through its centre.The ratio of the magnitudes of the magnetic moment of the system and its angular momentum about the centre of the rod is $\frac{kq}{4m}$ then k =

A) 1 B) 2 C) 3 D) 4

Correct Answer: B

Solution: current i = frquency $\times$ charge = $\frac{\omega}{2\pi} \times 2q$

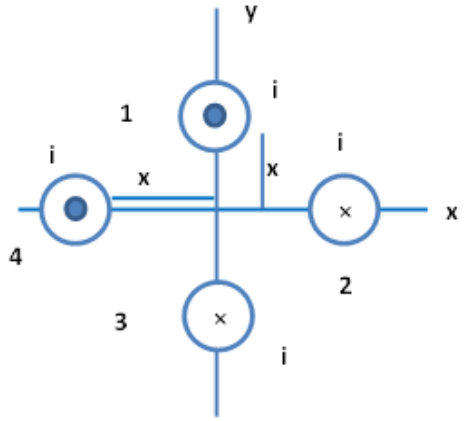
$\Rightarrow i = \frac{2q\omega}{2\pi} = \frac{q\omega}{\pi}$

Magnetic moment $M = iA = \frac{q\omega}{\pi}\pi R^2 = q\omega R^2$

Angular momentum $L = 2R.\, mv = 2R.\, mR\omega = 2mR^2\omega \;\; (v = R\omega)$

$\Rightarrow \frac{M}{L} = \frac{q}{2m}$

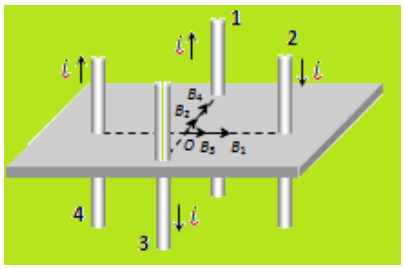
49) What will be the resultant magnetic field at origin due to four infinite length wires. If each wire produces magnetic field 'B' at origin is $\sqrt{k}B$ then $k =$



A) 4 B) 2 C) 8 D) 0

Correct Answer: C

Solution:



When a current flows through a conductor, it produces a magnetic field, shown in the above diagram as the green circular arrows surrounding the wire.

The direction of the current in the conductor is indicated by the symbols

⊗ which indicates a current flowing away from you, or INTO the plane of the paper,

and ? which indicates a current moving towards you, or OUT of the paper.

To get the direction of the magnetic field lines the Right-Hand Thumb Rule can be used.

Right-hand Thumb rule states that, if the thumb of the right hand is in the direction of the current flow then, the curl fingers show the direction of the magnetic field.

One should know that where we have to find magnetic field ?

since from point to point direction of magnetic field direction changes.

In this problem we have to find magnetic field at origin.

Other thing we required is what are the sources which provide magnetic field ?

Here current carrying conductors are the source for magnetic field.

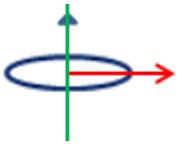
To find direction of magnetic field, we must know the direction of magnetic field.

That is clearly mentioned in this problem.

Now let us apply the right hand thumb rule to find the direction of magnetic field.

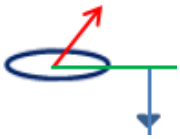
In all cases red mark shows the direction of magnetic field.

Case 1: At number 1



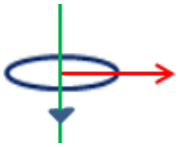
The direction of the current at 1 is out of the paper or perpendicular to the plane, then direction of magnetic field is due east.

Case 2 : At number 2



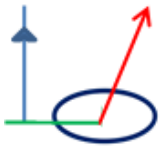
The direction of the current at 2 is into the paper or perpendicular to the plane, then direction of magnetic field is due north.

Case 2 : At number 3



The direction of the current at 3 is into the paper or perpendicular to the plane, then direction of magnetic field is due east.

Case 4 : At number 4



The direction of the current at 4 is perpendicular to the plane, then direction of magnetic field is due north.

Due to two wires 1, 3 direction of magentic field is to wards north.

Due to remaining two wires 2, 4 direction of magentic field is to wards East.

All the wires are placed at equal distance x from the origin and Magnetic induction due to infinite length wire at origin is $\frac{\mu_o i}{2\pi x}$

$\Rightarrow B_1 = B_2 = B_3 = B_4 = \frac{\mu_o i}{2\pi x} = B$

B_1, B_3 are in north direction their resultant is $(B_1 + B_3) = 2Bj$

B_2, B_4 are in east direction their resultant is $(B_2 + B_4)i = 2Bi$

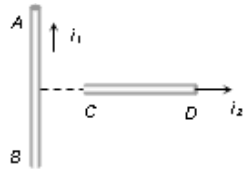
Resultant magentic field $\bar{B} = (B_2 + B_4)i + (B_1 + B_3)j = 2B(i + j)$

Magnitude of the resultant is$2\sqrt{2}B$

50) An infinitely long, straight conductor AB is fixed and a current is passed through it.

Another movable straight wire CD of finite length and carrying current is held perpendicular to

it and released. Neglect weight of the wire



A) The rod CD will move upwards parallel to itself B) The rod CD will move downward parallel to itself

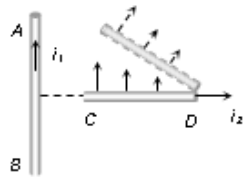
C) The rod CD will move upward and turn clockwise at the same time

D) The rod CD will move upward and turn anti –clockwise at the same time

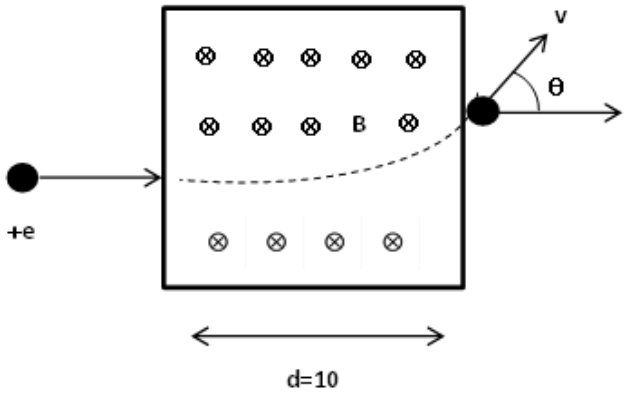
Correct Answer: C

Solution: Since the force on the rod CD is non-uniform it will experience force and torque. From

the left hand side it can be seen that the force will be upward and torque is clockwise



51) A proton accelerated by a potential difference $500\ KV$ moves though a transverse magnetic field of $0.51\ T$ as shown in figure. The angle θ^o through which the proton deviates from the initial direction of its motion is then θ



A) 15 B) 30 C) 45 D) 60

Correct Answer: B

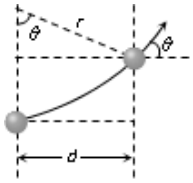
Solution: According to following figure $\sin\theta = \frac{d}{r}$

also,$r = \frac{\sqrt{2mk}}{qB} = \frac{1}{B}\sqrt{\frac{2mV}{q}}$

$\sin\theta = Bd\sqrt{\frac{q}{2mV}}$

$= 0.51 \times 0.1\sqrt{\frac{1.6\times10^{-19}}{2\times1.67\times10^{-27}\times500\times10^3}}$

$= \frac{1}{2} \Rightarrow \theta = 30^o$



52) Two identical short bar magnets, each having magnetic moment M, are placed a distance of 2d apart with axes perpendicular to each other in a horizontal plane. The magnetic induction at a point midway between them is

A) $\frac{\mu_0}{4\pi}(\sqrt{3})\frac{M}{d^3}$ B) $\frac{\mu_0}{2\pi}(\sqrt{3})\frac{M}{d^3}$ C) $\left(\frac{2\mu_0}{\pi}\right)\frac{M}{d^3}$ D) $\frac{\mu_0}{4\pi}(\sqrt{5})\frac{M}{d^3}$

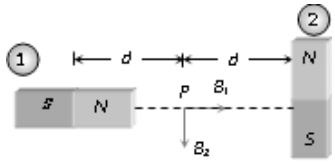
Correct Answer: D

Solution: At point P net magnetic field $B_{net} = \sqrt{B_1^2 + B_2^2}$

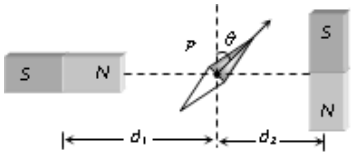
where $B_1 = \frac{\mu_0}{4\pi}.\frac{2M}{d^3}$ and

$B_2 = \frac{\mu_0}{4\pi}.\frac{M}{d^3}$

$\Rightarrow B_{net} = \frac{\mu_0}{4\pi}.\frac{\sqrt{5}M}{d^3}$



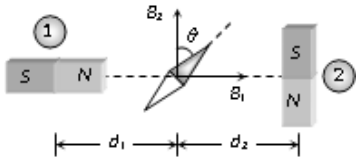
53) Two magnets A and B are identical and these are arranged as shown in the figure. Their length is negligible in comparison to the separation between them. A magnetic needle is placed between the magnets at point P which gets deflected through an angle θ under the influence of magnets. The ratio of distance d_1 and d_2 will be



A) $(2\tan\theta)^{1/3}$ B) $(2\tan\theta)^{-1/3}$ C) $(2\cot\theta)^{1/3}$ D) $(2\cot\theta)^{-1/3}$

Correct Answer: C

Solution: In equilibrium $B_1 = B_2\tan\theta$



$\Rightarrow \frac{\mu_0}{4\pi}.\frac{2M}{d_1^3} = \frac{\mu_0}{4\pi}.\frac{M}{d_2^3}\tan\theta$

$\Rightarrow \frac{d_1}{d_2} = (2\cot\theta)^{1/3}$

54) A copper rod of length l is rotated about one end perpendicular to the magnetic field B with constant angular velocity ω. The induced e.m.f. between thetwo ends is

A) $\frac{1}{2}B\omega l^2$ B) $\frac{3}{4}B\omega l^2$ C) $B\omega l^2$ D) $2B\omega l^2$

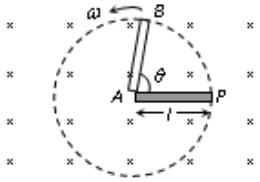
Correct Answer: A

Solution: If in time t. the rod turns by an angle q, the area generated by the rotation of rod will be $= \frac{1}{2}l \times l\theta = \frac{1}{2}l^2\theta$.

So the flux linked with the area generated by the rotation of rod

$\phi = B\ \left(\frac{1}{2}l^2\theta\right)\cos 0 = \frac{1}{2}Bl^2\theta = \frac{1}{2}Bl^2\omega\, t$

And so$e = \frac{d\phi}{dt} = \frac{d}{dt}\left(\frac{1}{2}Bl^2\omega t\right) = \frac{1}{2}Bl^2\omega$



55) An e.m.f. of 15 volt is applied in a circuit containing 5 henry inductance and 10 ohm resistance. The ratio of the currents at time $t = \infty$ and at t = 1 second is

A) $\frac{e^{1/2}}{e^{1/2}-1}$ B) $\frac{e^2}{e^2-1}$ C) $1 - e^{-1}$ D) e^{-1}

Correct Answer: B

Solution: $i = i_0(1 - e^{-Rt/L})$

$i_0 = \frac{E}{R}$ (Steady current) when $t = \infty$

$i_\infty = \frac{E}{R}(1 - e^{-\infty}) = \frac{5}{10} = 1.5$

$i_1 = 1.5(1 - e^{-R/L}) = 1.5(1 - e^{-2})$

$\frac{i_\infty}{i_1} = \frac{1}{1-e^{-2}} = \frac{e^2}{e^2-1}$

56) Two coils have a mutual inductance 0.005 H. The current changes in the first coil according to equation $I = I_0 \sin\omega\, t$, where $I_0 = 0A$ and w = 100 π radian/sec. The maximum value of e.m.f. in the second coil is

A) 2π B) 5π C) π D) 4π

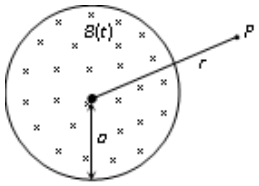
Correct Answer: B

Solution: $e = M\frac{di}{dt} = 0.005 \times \frac{d}{dt}(i_0 \sin\omega t)$

$= 0.005 \times i_0\omega\cos\omega\, t$

$e_{\max} = 0.005 \times 10 \times 100\pi = 5\pi$

57) A uniform but time-varying magnetic field B(t) exists in a circular region of radius a and is directed into the plane of the paper, as shown. The magnitude of the induced electric field at point P at a distance r from the centre of the circular region



A) is zero B) Decreases as $\frac{1}{r}$ C) Increases as r D) Decreases as $\frac{1}{r^2}$

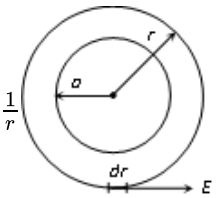
Correct Answer: B

Solution: Construct a concentric circle of radius r. The induced electric field (E) at any point on the circle is equal to that at P. For this circle

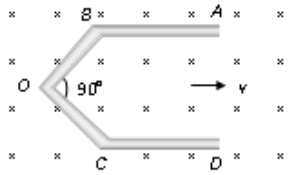
$$\oint \vec{E}.\vec{dl} = \left|\frac{d\varphi}{dt}\right| = A\ \left|\frac{dB}{dt}\right|$$

or $E \times (2\pi r) = \pi a^2 . \left|\frac{dB}{dt}\right|$

$$\Rightarrow E = \frac{a^2}{2r}\left|\frac{dB}{dt}\right| \Rightarrow E \propto \frac{1}{r}$$



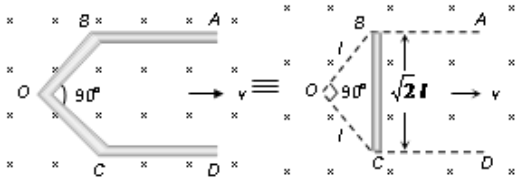
58) A conductor ABOCD moves along its bisector with a velocity of $1m/s$ through a perpendicular magnetic field of $1wb/m^2$, as shown in fig. If all the four sides are of 1m length each, then the induced emf between points A and D is



A) 0 B) 1.41 volt C) 0.71 volt D) None of these

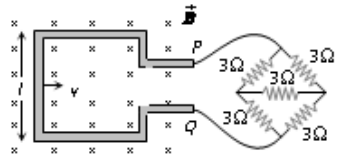
Correct Answer: B

Solution: There is no induced emf in the part AB and CD because they are moving along their length while emf induced between B and C i.e. between A and D can be calculated as follows



Induced emf between B and C = Induced emf between A and B =
$Bv(\sqrt{2}\,l) = 1 \times 1 \times 1 \times \sqrt{2} = 1.41\,volt.$

59) A square metallic wire loop of side 0.1 and resistance of 1Ω is moved with a constant velocity in a magnetic field of $2Wb/m^2$ as shown in figure. The magnetic field is perpendicular to the plane of the loop, loop is connected to a network of resistances. What should be the velocity of loop so as to have a steady current of $1mA$ in loop



A) $1cm/sec$ B) $2cm/sec$ C) $3cm/sec$ D) $4cm/sec$

Correct Answer: B

Solution: Equivalent resistance of the given Wheatstone bridge circuit (balanced) is 3Ω so total resistance in circuit is $R = 3 + 1 = 4\Omega$. The emf induced in the loop $e = Bvl$.

So induced current $i = \frac{e}{R} = \frac{Bvl}{R} \Rightarrow 10^{-3} = \frac{2 \times v \times (10 \times 10^{-2})}{4} \Rightarrow v = 2cm/sec.$

60) The true value of angle of dip at a place is 60^0 the apparent dip in a plane inclined at an angle of 30^0 with magnetic meridian is

A) $\tan^{-1}\frac{1}{2}$ B) $\tan^{-1}(2)$ C) $\tan^{-1}\left(\frac{2}{3}\right)$ D) None of these

Correct Answer: B

Solution: $\tan\varphi' = \frac{\tan\varphi}{\cos\beta}$; where φ' =Apparent angle of dip, φ = True angle of dip, β = Angle made by vertical plane with magnetic meridian.

$\Rightarrow \tan\varphi' = \frac{\tan 60^o}{\cos 30^o} = 2 \Rightarrow \varphi' = \tan^{-1}(2)$

61) Which one of the following is an antihistamine

A) Terpineol B) Chloramphenicol C) Zantac D) Salvarsan

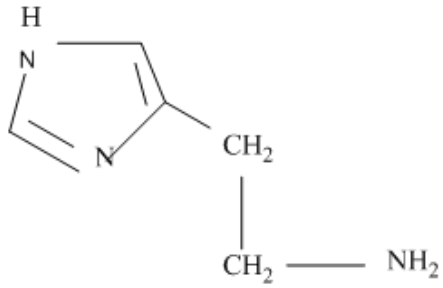
Correct Answer: C

Solution: Terineol $\rightarrow$Antisetic

Chloramhenicol$\rightarrow$Antibiotic

Zantac $(ranitidine) \rightarrow$ Antihistamine

Salvarsan$\rightarrow$ Antimicrobial

62) The drug  is used as

A) Antacid B) Analgesic C) Antimicrobial D) Antiseptic

Correct Answer: A
Solution: This drug is used as an antacid

63) Artificial sweetener which is stable under cold conditions only is

A) Aspartame B) Saccharine C) Alitame D) Sucralose

Correct Answer: A
Solution: Asartame decomposes at cooking temperature

64) The Hormone that helps in the conversion of glucose to glycogen is

A) Adrenaline B) Insulin C) Cortisone D) Bile acids

Correct Answer: B
Solution: Insulin is a hormone secreted by the pancreas that lower blood glucose levels by promoting the uptake of glucose by cells and the conservation of glucose to glycogen by the liver and skeletal muscle.

65) Which one of the following bases is not present in DNA

A) Quinoline B) Adenine C) Cytosine D) Thymine

Correct Answer: A
Solution: Quinoline is an alkaloid,it is not present in DNA,DNA has four nitrogenous bases adenine ,guanine,cytosine and thymine

66) Which of the set consists only of essential amino acids

A) Alanine,tyrosine,cystine B) Leucine lysine,tryptophan C) Alanine,glutamine,lysine D) Leucine,proline,glycine

Correct Answer: B
Solution: Essential amino acids are those amino acids which are supplied to our bodies by food because they

cannot be synthesized in the body Essential amino acids are listed as;Valine,Leucine

,Isoleucine,Phenyl-alanineTryptophan Threonine Methionine Lysine,Arginine and Histidine

67) Which of the statements about “Denaturation “given below are correct

(A) Denaturation of proteins causes loss of secondary and tertiary structures of the protein

(B)Denaturation leads to the conversion of double strand of DNA into single strand

(C)Denaturation affects primary structure which gets distorted

A) (A) and (B) B) (A) (B) and (C) C) (B) and (C) D) (A) and (C)

Correct Answer: A
Solution: During denaturation secondary and tertiary structures of protein destroyed but primary structures remains intact

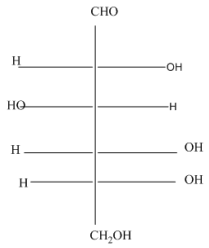
68) Which of the following pairs give positive Tollen’s test

A) Glucose,sucrose B) Glucose ,fructose C) Hexanal acetophenone D) fructose,sucrose

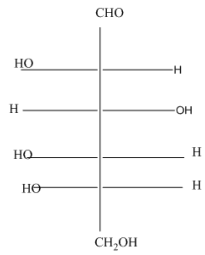
Correct Answer: B
Solution: Aldehydes and $\alpha-$ hydroxyl ketones give positive Tollen’s test Glucose has an aldehyde

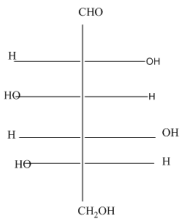
group and fructose is an $\alpha-$hydroxyl ketone

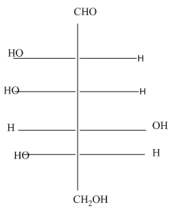
69) The structure of $D-(+)-$ glucose is

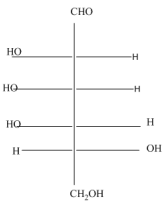


The structure of $L-(-)-$glucose is

A)


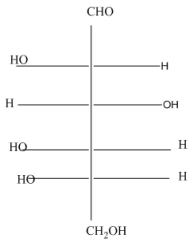
B)


C)


D)


Correct Answer: A

Solution: The structure of $L-(-)-$glucose is



70) Among cellulose polyvinyl chloride nylon and natural rubber ,the polymer in which the intermolecular force of attraction is weakest in

A) Nylon B) Polyvinyl chloride C) Cellulose D) Natural rubber

Correct Answer: D

Solution: The natural rubber has intermolecular forces which are weak disersion force vander Waals forces of attraction and is an example of an elastomer polymer

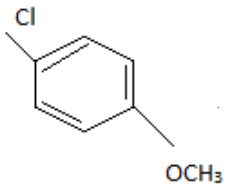
71) Ebonite is

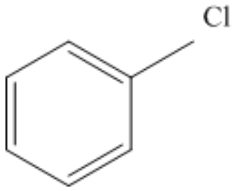
A) Polropene B) Natural rubber C) Synthetic rubber D) Highly vulcanized

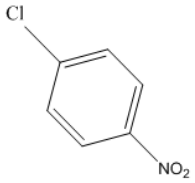
Correct Answer: D

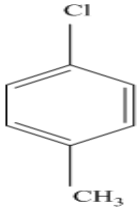
Solution: Ebonite is a hard and highly 20-30% vulcanized rubber

72) Which of the following compounds undergoes nucleophilic substitution reaction most easily

A) 

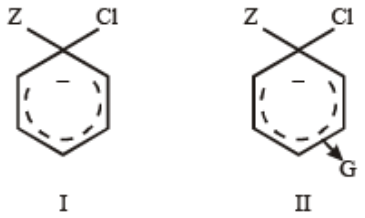
B) 

C) 

D) 

Correct Answer: C

Solution: In SNAr reactions, a carbanion is formed as an intermediate, so any substituent that increases the stability of carbanion and hence the transition state leading to its formation will enhance the SNAr reactions. To compare the rates of substitution in chlorobenzene, chlorobenzene having electron-withdrawing group, and chlorobenzene having electronreleasing group, we compare the structures carbanion I from chlorobenzene, II from chlorobenzene containing electron-withdrawing group and III from chlorobenzene containing electron-releasing group.



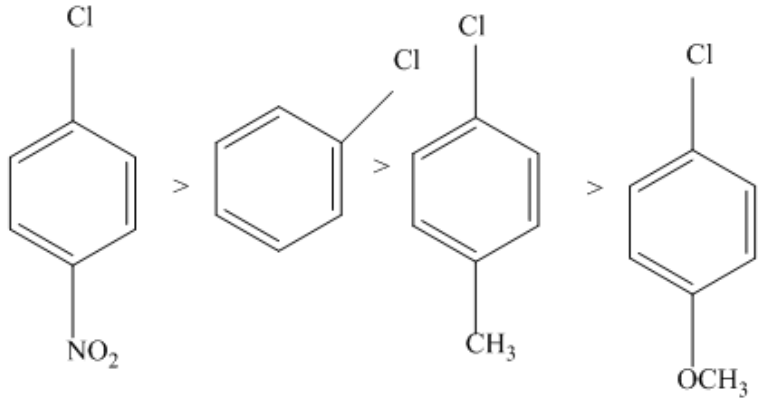
I II

G withdraws electrons, neutralises negative charge of the ring, stabilises carbanion, facilitates SN reaction $(activation effect)$

III

G releases electrons, intensifies –ve charge, destabilizes carbanion, retards SN reaction $(deactivation) NO_2$ is activating group and CH_3 and OCH_3 are deactiving group.

Hence, the correct order of nucleophilic substitution reactions



73) Identify Z in the following series

$$C_2H_5I \xrightarrow{Alco.KOH} X \xrightarrow{Br_2} Y \xrightarrow{KCN} Z$$

A) CH_3CH_2CN B) CH_2CN / CH_2CN C) $BrCH_2 - CH_2CN$ D) $BrCH = CHCN$

Correct Answer: B

Solution: $C_2H_5I \xrightarrow{alc.KOH} C_2H_4 \xrightarrow{Br_2}$ $CH_2(Br)-CH_2(Br)$

$\xrightarrow{KCN}$ $CH_2(CN)-CH_2(CN)$

Butane -1, 4-dinitrile

74) Unknown alcohol is treated with the “Lucas reagent” to determine whether the alcohol is primary secondary or tertiary. Which alcohol reacts fastest and by what mechanism

A) Secondary alcohol by S_N^1 B) Tertiary alcohol by S_N^1 C) Secondary alcohol by S_N^2

D) Tertiary alcohol by S_N^2

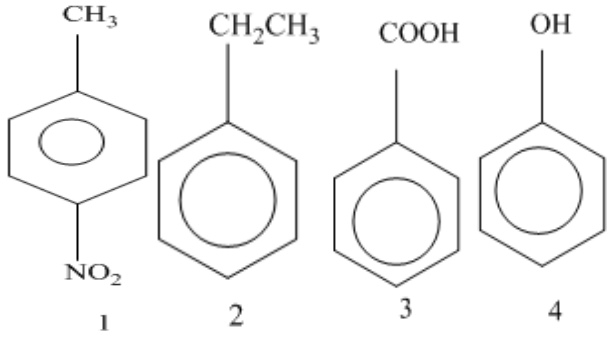
Correct Answer: B

Solution: Lucas test is a test to recognize whether the given alcohol is primary, secondary, or tertiary in nature. Lucas reagent is a mixture of concentrated. HCl and dry anhydrous $ZnCl_2$.

In this reaction, a carbocation is formed as an intermediate and thus the stability and ease of carbocation formation determine the rate of the reaction as that's the slow step of the mechanism. Hence the order of reactivity is tertiary alcohol > secondary alcohol by S_N1 mechanism.

Thus with Lucas reagent, tertiary alcohol reacts fastest and by S_N1 mechanism.

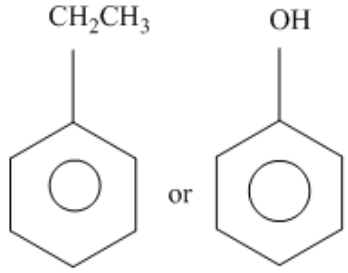
75) Which will undergo a Friedel-Craft’s alkylation reaction



A) 1,2 and 4 B) 1 and 3 C) 2 and 4 D) 1 and 2

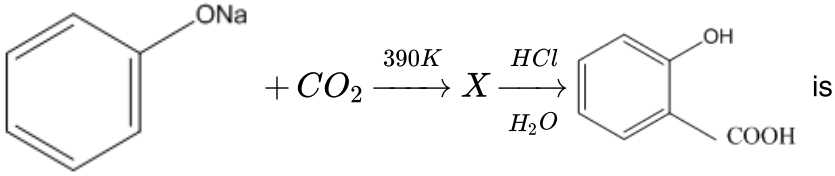
Correct Answer: C

Solution:

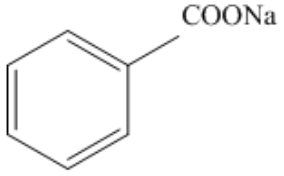


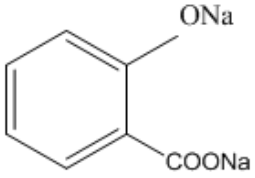
will undergoes a Friedel Craft’s alkylation on para position because of more electron density.

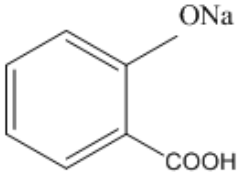
76) The compound X in the reaction

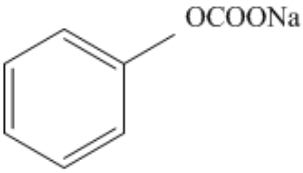


is

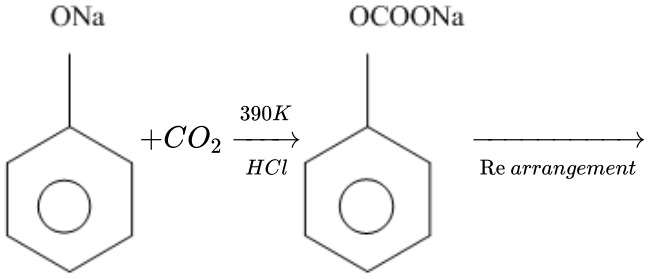
A) 

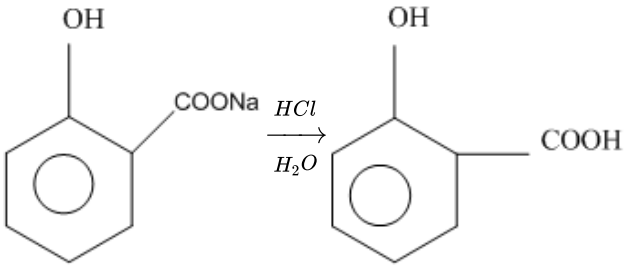
B) 

C) 

D) 

Correct Answer: D

Solution: 

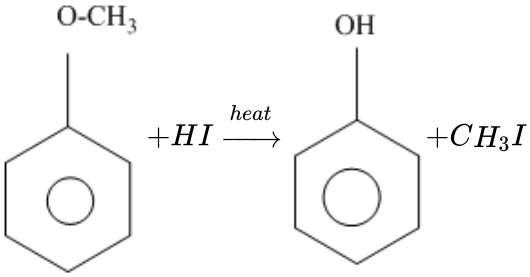


77) The products formed in the following reaction

$C_6H_5 - O - CH_3 + HI \xrightarrow{heat} are$

A) $C_6H_5 - I$ and $CH_3 - OH$ B) $C_6H_5 - OH$ and $CH_3 - I$ C) $C_6H_5 - CH_3$ and HOI
D) C_6H_6 and CH_3OI

Correct Answer: B

Solution: 

78) Which of the following pairs can be distinguished by sodium hypoiodite

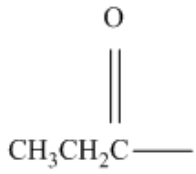
A) CH_3CHO and CH_3COCH_3 B) CH_3CH_2CHO andCH_3COCH_3

C) CH_3CH_2OH and$CH_3CH_2CHOHCH_3$ D) CH_3OH andCH_3CH_2CHO

Correct Answer: B

Solution: Aldehydes and ketones having at least one methyl group linked to carbonyl

atom gives this test So,here in option(b) ketone is having (CH_3CO-) group andthe other is having



group which do not give hypoidite test.So thus they can be distingushed

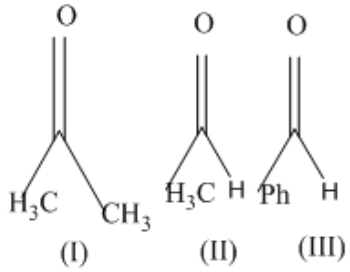
79) The key step in cannizzaro's reaction is the intermolecular shift of

A) Proton B) Hydride ion C) Hydronium ion D) Hydrogen bond

Correct Answer: B

Solution: Cannizzaro reaction is an example of hydride ion (H^-)transfer reaction

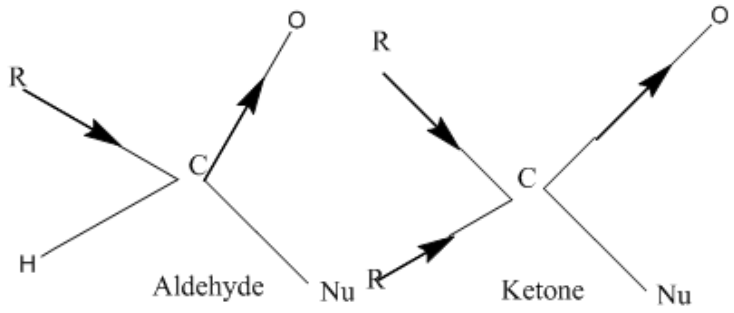
80) The order or reactivity of phenyl magnesium bromide with the following compound is



A) (II)>(III)>(I) B) (I)>(III)>(II) C) (II)>(I)>(III) D) All react with the same rate

Correct Answer: C

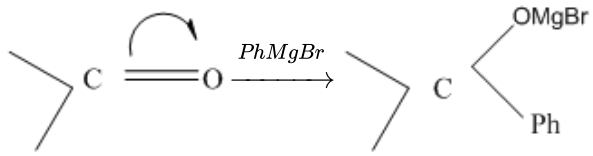
Solution: In phenyl magnesium bromide $\left(\overset{\delta-}{Ph}\overset{\delta+}{Mg}Br\right)^{\delta-}$ Ph is attached with that C-atom of carbonyl group which have low electron density(higher electropositive charge).In carbonyl compounds, aldehydes are more reactive



Towards nucleophile in nucleophilic addition reaction because in ketonesalkyl groups(due to +1 effect)decrease the electropositive charge of carbon of carbonyl group. Hence attraction of nuclephilicdecreases.Morever in the tetrahedral intermediate aldehyde have less steric repulsion than

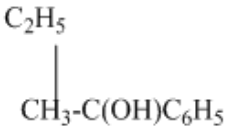
ketones and also the aldehyde increases the negative charge on oxygen less in comparison of ketones.

Thus, on the basis of above reason the order of reactivity of acetones(1),acetaldehyde(II) and benzaldehyde(III) with PhMgBr is II > I> III



81) In a set of the given reactions, acetic acid yielded a product C

$CH_3COOH + PCl_5 \rightarrow A \xrightarrow[anh.AlCl_3]{C_6H_6} B \xrightarrow[ether]{C_2H_5MgBr} C$ Product C would be

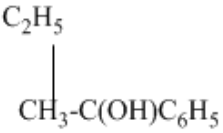
A) 

B) $CH_3CH(OH)C_2H_5$ C) $CH_3COC_6H_5$

D) $CH_3CH(OH)C_6H_5$

Correct Answer: A

Solution: $CH_3COOH + PCl_5 \rightarrow CH_3COCl \xrightarrow[anh.AlCl_3]{C_6H_6}$

$CH_3COC_6H_5 \xrightarrow[ether]{C_2H_5MgBr}$ 

82) Which of the following acids has the smallest dissociation constant

A) $CH_3CHFCOOH$ B) FCH_2CH_2COOH C) $BrCH_2CH_2COOH$

D) $CH_3CHBrCOOH$

Correct Answer: C

Solution: $BrCH_2CH_2COOH$ is least acidic or has less K_a weak is acid

i.e dissociation constant It is due to lesser –I effect of Br than F and Br atom further away form $-COOH$ group

acetic acid has the lowest inductive effect of attached bromide due to long distance and low electronegativity of Br as compared to other acids where the halide is comparatively nearer or stronger.

F is more electronegative than Br so more -I effect due to F.

Inductive effect: The effect on electron density in one portion of a molecule due to electron-withdrawing or electron-donating groups elsewhere in the molecule.

83) Acetamide is treated separately with the following reagents.Which would give methyl amine

A) PCl_5 B) $NaOH + Br_2$ C) Sodalime D) Hot concH_2SO_4

Correct Answer: B

Solution: $\underset{Acetamide}{CH_3CONH_2} + Br_2 + 4NaOH \rightarrow$

$\underset{Methyla\min e}{CH_3NH_2} + Na_2Co_3 + 2NaBr + 2H_2O$

84) $CH_3NH_2 + CHCl_3 + KOH \rightarrow$ Nitrogen containing compound $+KCl + H_2O.$ Nitrogen containing compound is

A) $CH_3 - C \equiv N$ B) $CH_3 - NH - CH_3$ C) $CH_3 - N \equiv C^+$ D) $CH_3N^+ \equiv C^-$

Correct Answer: D

Solution: The reaction of primary amine $R - NH_2$ with chloroform $CHCl_3$ and a base (KOH) is known as carbylamine reaction. The reaction yields isocyanide as the product, along with salt and water.

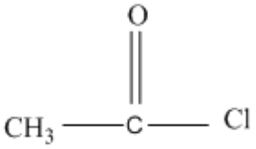
$CH_3NH_2 + CHCl_3 + KOH \rightarrow CH_3N^+ \equiv C^- + KCl + H_2O$

85) A compound with molecular mass 180 is acylated with CH_3COCl to get a compound with molecular

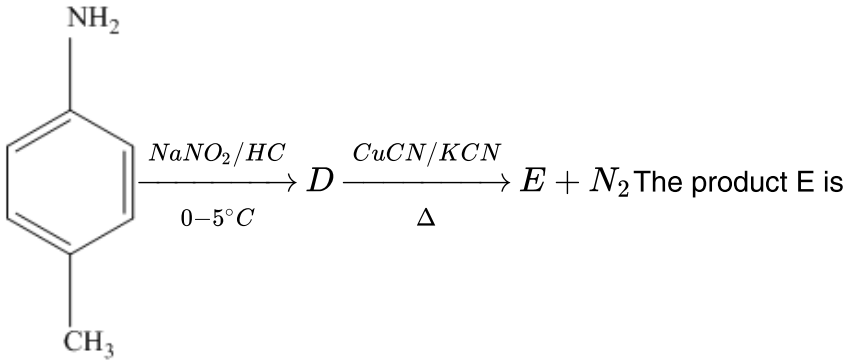
mass 390 The number of amino groups present per molecule of the former compound is

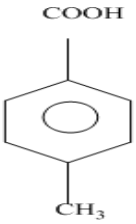
A) 2 B) 5 C) 4 D) 6

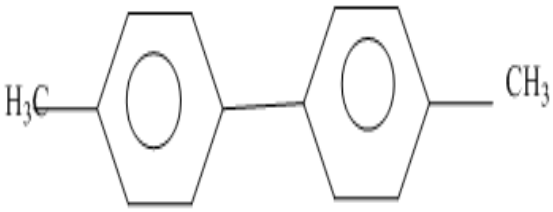
Correct Answer: B

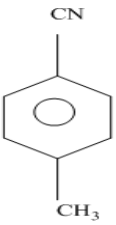
Solution: By reaction with one mole of  with one NH_2 group the molecular mass

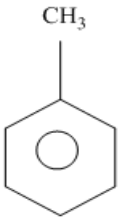
increases with 42 unit. since the mass increases by (390-180)=210,hence the no.of NH_2 group are 5.

86) In the reaction  The product E is

A) 

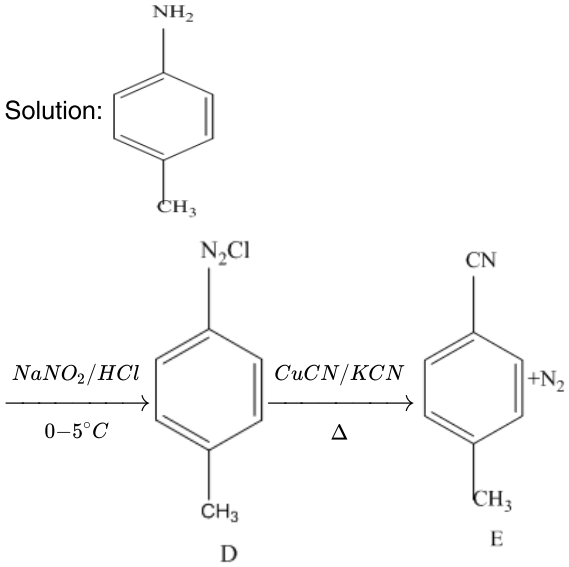
B) 

C) 

D) 

Correct Answer: C

Solution:



87) The incorrect statement among the following is

A) Hydrogen is used to reduce NiO B) Zirconium is refined by van Arkel method

C) The sulphide ore galena is concentrated by froth floatation

D) In the metallurgy of iron ,the flux used is SiO_2

Correct Answer: D

Solution: CaO acts a flux as it combines with silica present as an impurity to form a fusible slag, $CaSiO_3$

88) Considering Ellingham diagram ,which of the following metals can be used to reduce alumina

A) Fe B) Zn C) Mg D) Cu

Correct Answer: C
Solution: Mg has more $-\Delta G$value then alumina so it will be in the lower part of Ellingham diagram

Metals which has more $-\Delta G$value can reduce those metals oxide which has less $-\Delta G$value

89) Roasted gold ore $+CN^- + H_2O \xrightarrow{O_2} [X] + OH^-; [X] + Zn \rightarrow [Y] + Au\,[X]$ and $[Y]$ are

A) $X = [Au(CN)_2]^-$; $Y = [Zn(CN)_4]^{2-}$

B) $X = [Au(CN)_4]^{3-}$ $Y = [Zn(CN)_4]^{2-}$

C) $X = [Au(CN)_2]^-$ $Y = [Zn(CN)_6]^{4-}$

D) $X = [Au(CN)_4]^{3-}$ $Y = [Zn(CN)_6]^{2-}$

Correct Answer: A
Solution: $2Au + 4CN^- + H_2O + \frac{1}{2}O_2 \rightarrow 2Au(CN)_2^- + 2OH^-$

$2[Au(CN)_2]^- + Zn \rightarrow [Zn(CN)_4]^{2-} + 2Au$

90) Which one of the following ores is best concentrated by froth floatation method

A) Siderite B) Galena C) Malachite D) Magnetite

Correct Answer: B
Solution: Galena=PbS

For sulphur ores froth floatation is carried out.

91) Excess pressure inside a soap bubble is three times that of the other bubble, then the ratio of their volumes will be

A) 1:3 B) 1:9 C) 1:27 D) 1:81

Correct Answer: C
Solution: P_i = Pressure inside the soap bubble.

P_o = Pressure outside the soap bubble.

Excess pressure inside the soap bubble $= \Delta P = P_i - P_o = \frac{\Delta T}{r}$

$\Delta P = \frac{\Delta T}{r}$

$\Rightarrow \Delta P \propto \frac{1}{r}$

$\Rightarrow \frac{\Delta P_1}{\Delta P_2} = \frac{r_2}{r_1} \Rightarrow \frac{r_2}{r_1} = \frac{3}{1}$

Soap buble is in sphereical shape, hence volume of it is equal to

$V = \frac{4\pi r^3}{3}$

$\Rightarrow \frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$

92) The least gold number is possible for

A) Potato starch B) Maize starch C) Rice starch D) gelatin

Correct Answer: D
Solution: Gelatin has least gold number 0.005 to 0.01

93) The values 'a' for which the function $f(x) = (a+2)x^3 - 3ax^2 + 9ax - 1$ decreases for all real values of x is

A) $a \leq -2$ B) $a \geq -2$ C) $a \leq -3$ D) $-3 \leq a < -2$

Correct Answer: C
Solution: f decreases for all $x \Rightarrow f'(x) \leq 0 \quad \forall x \in R$

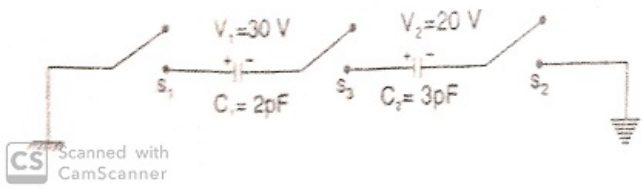
$\Rightarrow (a+2)x^2 - 2ax + 3a \leq 0.$

$\Rightarrow 4a^2 - 4(a+2)(3a) \leq 0 \; and \; a+2 < 0 \Rightarrow a^2 - (a+2)(3a) \leq 0$

$\Rightarrow -2a^2 - 6a \leq 0 \Rightarrow a^2 + 3a \geq 0 \Rightarrow a \leq 0 \; and \; a \geq -3$

Hence $-3 \leq a < -2.$

94) For the circuit shown, which of the following statement is true



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A) With S_1 closed $V_1 = 15V, V_2 = 20V$ B) With S_3 closed $V_1 = V_2 = 25V$

C) With S_1 and S_2 closed $V_1 = V_2 = 0$ D) With S_1 and S_3 closed $V_1 = 30V, V_2 = 20V$

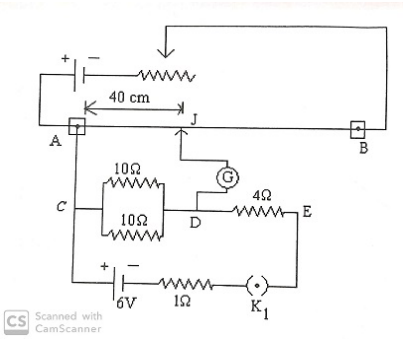
Correct Answer: D
Solution:

95) A parallel plate capacitor of capacitance C is connected to a battery and is charged to a potential difference V. Another capacitor of capacitance 2C is similarly charged to a potential difference 2V. The charging battery is now disconnected and the capacitors are connected in parallel to each other in such a way that the positive terminal of one is connected to the negative terminal of the other. The final energy of the configuration is

A) zero B) $\frac{3}{2}CV^2$ C) $\frac{35}{6}CV^2$ D) $\frac{9}{2}CV^2$

Correct Answer: B
Solution:

96) In the circuit shown in fig., the potential difference between the points C and D is balanced against 40 cm length of potentiometer wire of total length 100 cm. In order to balance the potential difference between the points D and E. The jockey be pressed on potentiometer wire at a distance of in cm



A) 16 B) 32 C) 56 D) 80

Correct Answer: B
Solution:

97)
Breathing movements can be estimated by

A) Barometer B) Spirometer C) Spigmomanometer D) Voltmeter

Correct Answer: B
Solution: Spirometry is the process of recording the changes in the volume and movement of air in and out of the lungs and the instrument used for this purpose is called spirometer or respirometer.