RB IIT Academy EAMCET ENG MOCK TEST 2

1) The domain of $f(x) = Cot(2x-4)$ is

A) $R - \{n\pi : n \in N\}$ B) $R - \left\{\frac{n\pi+4}{2};\ n \in Z\right\}$ C) $R - \left\{\frac{n\pi}{5};\ n \in Z\right\}$ D) $R - \left\{(2n+1)\frac{\pi}{10};\ n \in Z\right\}$

Correct Answer: B
Solution: $Cot(2x-4)$ is defined if $\{x/2x-4 \neq n\pi, n\ \in Z\}$
i.e $\{x/x \neq \frac{n\pi+4}{2}, n\ \in Z\}$

2) The range of $x^2+4y^2+9z^2-6yz-3xz-2xy$

A) ϕ B) R C) $[0,\infty)$ D) $(-\infty,0)$

Correct Answer: C
Solution: $x^2+4y^2+9z^2-6yz-3xz-2xy = \frac{1}{2}(2x^2+8y^2+18z^2-12yz-6xz-4xy)$
$= (x-2y)^2+(2y-3z)^2+((3z-x)^2 \geq 0$
Hence Range of given function is $[0,\infty)$

3) The points with vectors $60\bar{i}+3\bar{j}, 40\bar{i}-8\bar{j}\ and\ \ a\bar{i}-52\bar{j}$ are collinear, if $'a' =$

A) -40 B) 40 C) 20 D) 30

Correct Answer: A
Solution: $let\, \overline{OA} = 60\bar{i}+3\bar{j},\ \overline{OB} = 40\bar{i}-8\bar{j}\ and\ \ \overline{OC} = a\bar{i}-52\bar{j}.$
A,B,C are collinear $\Rightarrow \overline{AB} = K\overline{BC}\ \Rightarrow -20\bar{i}-11\bar{j}\ = K((a-40)\bar{i}-44\bar{j}\ )$
Comparing i, j coefficients $-44k = -11$
$\Rightarrow k = \frac{1}{4} \Rightarrow \frac{1}{4}(a-40) = -20 \Rightarrow a = -40$

4) If A = $\bar{a}$ and B = $\bar{b}, C = 2\bar{a}-4\bar{b}$ then

A) C lies outside ΔOAB but inside exterior angular region of $\angle OBA$.
B) C lies outside ΔOAB but inside $\angle OAB$. C) C lies outside ΔOAB but inside $\angle AOB$.
D) C lies inside ΔOAB but outside $\angle ABO$.

Correct Answer: A
Solution: $\bar{c} = l\bar{a}+m\bar{b}, l>0, m<0 \Rightarrow \bar{c}$ lies out side the triangle OAB
If $|l|+|m|<1$ the point $C(\bar{c})$ lies in side the $\angle OBA$, but here

$|l|+|m|=|2|+|-4|=6>1$ then it lies exterior angular region of B.

5) A vector has components $2p$ and 1 w.r.t. a rectangular cartesian system.This system is roatated through a certain angle about the origin in the counter -clockwise with respect to the new system, if has components $p+1$ and 1 then

A) $p=0$ B) $p=1$ or $p=-\frac{1}{3}$ C) $p=-1$ or $p=\frac{1}{3}$ D) $p=1$ or $p=-1$

Correct Answer: B

Solution: counter clock wise = anticlock wise

$\bar{r}=2pi+j.$

If system is rotated about the origin then vector in the new position is $\bar{r}=(p+1)i+j$ there is no change in the magnitude ,hence

$\sqrt{4p^2+1}=\sqrt{(p+1)^2+1}\Rightarrow 2p=p+1$or $2p=-(p+1)\Rightarrow p=1,p=\frac{-1}{3}$

Vector in the new positions are either $\bar{r}=2i+j$ or $\bar{r}=\frac{2i}{3}+j.$

If p= 1 then there is no change in the vector.

Hence possible vector is $\bar{r}=\frac{2i}{3}+j.$

6) If $7\overline{a}+3\overline{b}-10\overline{c}=\overline{0}$, then the vectors $\overline{a},\ \overline{b},\ \overline{c}$ are

A)Non-collinear and Non-coplanar B)Non-coplanar C)Coplanar D)Collinear

Correct Answer: C

Solution: Scalars are not all zero $\Rightarrow\ \overline{a}\ \overline{b},\ \overline{c}$ are linearly dependent.

$\Rightarrow$ One vector can be written as combination of other vectors.

Hence given vectors are coplanar vectors.

7) If $2\bar{i}+3\bar{j}+5\bar{k}$, $2\bar{i}+4\bar{j}+5\bar{k}$ and $2\bar{i}+4\bar{j}+7\bar{k}$ are the vertices of a triangle, then its circumcentre is

A) $2\bar{i}+\frac{11}{3}\bar{j}+\frac{17}{3}\bar{k}$ B) $2\bar{i}+\frac{7}{2}\bar{j}+6\bar{k}$ C) $6\bar{i}+15\bar{j}+17\bar{k}$ D) $2\bar{i}-16\bar{j}+18\bar{k}$

Correct Answer: B

Solution: Let $\overline{OA},\ \overline{OB},\ \overline{OC}$ Represents the position vectoors of vertices of triangle ABC

Let $\overline{OA}=2\bar{i}+3\bar{j}+5\bar{k}$

$\overline{OB}=2\bar{i}+4\bar{j}+5\bar{k}$

$\overline{OC}=2\bar{i}+4\bar{j}+7\bar{k}$

$\overline{AB}=\overline{OB}-\overline{OA}=\bar{j}$

$\overline{BC}=\overline{OC}-\overline{OB}=2\bar{k}$

$\Rightarrow \overline{BC}\perp\overline{AB}$

Hence circumcentre will be midpint of Hypotenuse AC

Position vector of circumcentre is $\frac{\bar{a}+\bar{c}}{2} = 2i + \frac{7j}{2} + 6k$

8) Combined equation of pair of lines passing through origin and inclined at 30^0 and 60^0 respectively with x - axis is

A) $\sqrt{3}y^2 - x^2 = 0$ B) $y^2 - \sqrt{3}x^2 = 0$ C) $\sqrt{3}y^2 + 4xy - \sqrt{3}x^2 = 0$

D) $\sqrt{3}y^2 - 4xy + \sqrt{3}x^2 = 0$

Correct Answer: D

Solution: Equation of line through origin $y = tan30^o x, y = tan60^o x$

$y = \frac{1}{\sqrt{3}}x, y = \sqrt{3}x \Rightarrow (\sqrt{3}y - x)(y - \sqrt{3}x) = 0$

$\Rightarrow \sqrt{3}y^2 - 4xy + \sqrt{3}x^2 = 0$

9) ΔABC is not a right angled and is inscribed in a fixed circle . If a A,b,B be slightly varied keeping c, C fixed then $\frac{\delta a}{\cos A} + \frac{\delta b}{cosB} =$.

A) 2 B) 1 C) 0 D) 5

Correct Answer: C

Solution: $A + B + C = 180$

$a = 2R\sin A, b = 2R\sin B$

$\Rightarrow \delta a = 2R\cos A\,\delta A\; and\; \delta b = 2R\cos B\delta B$

$A + B + C = 180 \Rightarrow \delta A + \delta B = 0$

$\Rightarrow \frac{\delta a}{\cos A} + \frac{\delta b}{cosB} = 0$

10) If A is a 3 x 3 skew-symmetric matrix, then trace of A is equal to

A) `-1` B) `1` C) $|A|$ D) 0

Correct Answer: D

Solution: A matrix is said to be skeW symmetric if $A^T = -A$

i.e $\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}^T = -\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$

$\Rightarrow a = -a, e = -e, f = -f \Rightarrow a = 0, e = 0, f = 0 \Rightarrow a + e + f = 0$

$\Rightarrow$ trace(A) = 0

11) If the entries in a 3×3 determinant are either 0 or 1, then the greatest value of their determinats is

A) 1 B) 2 C) 3 D) 9

Correct Answer: B

Solution: To get gratest value of the determinant with the entries Keep least of given values in principal diagonal, highest of given values in other places.

$$\begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} = 2.$$

12) A man is walking at the rate of 8 kmph towards the foot of the tower 60 mts high .The rate at which he approaches the top of the tower when he is 80 mts. from the foot of the tower is

A) 3.2 kmph B) 4.8 kmph C) 6.4 kmph D) 2.3 kmph

Correct Answer: B

Solution: z = 60 mt, x = 80 mt, y = 100 mt. $x^2 + 60^2 = y^2$

$2x\frac{dx}{dt} = 2y\frac{dy}{dt}$

Speed of the man = $\frac{dx}{dt} = 8kmph$

$\Rightarrow \frac{dy}{dt}\ \frac{60\times 8}{100} = 4.8kmph$

13) The slope of the tangent to the curve $x = t^2 + 3t - 8, y = 2t^2 - 2t - 5$ at the point $(2, -1)$ is

A) $\frac{22}{7}$ B) $\frac{6}{7}$ C) 6 D) -6

Correct Answer: B

Solution: $t^2 + 3t - 8 = 2$ and $2t^2 - 2t - 5 = -1$.Solving these equations we get $t = 2$.

Slope of tangent to the curve $\frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt} = \frac{4t-2}{2t+3} = \frac{6}{7}$

14) The angle between the planes $2x - y + z = 6\, and\, x + y + 2z = 7$ is

A) 30^0 B) 60^0 C) 45^0 D) 90^o

Correct Answer: B

Solution: The angle between the planes $a_1x + b_1y + c_1z + d_1 = 0, a_2x + b_2y + c_2z + d_2 = 0$ is θ then

$Cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\times\sqrt{a_2^2+b_2^2+c_2^2}}$

$Cos\theta = \frac{2-1+2}{\sqrt{6}\times\sqrt{6}} = \frac{3}{6} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

15) If $(\ell_1, m_1, n_1), (\ell_2, m_2, n_2)$ are dc's of two lines then
$(\ell_1 m_2 - \ell_2 m_1)^2 + (m_1 n_2 - m_2 n_1)^2 + (n_1 \ell_2 - n_2 \ell_1)^2 + (\ell_1 \ell_2 + m_1 m_2 + n_1 n_2)^2$

A) 0 B) 1 C) 2 D) 4

Correct Answer: B

Solution: $If\,\bar{a}\,=l_1i+l_2j+l_3k,\,\bar{b}\,=m_1i+m_2j+m_3k$ then

$\bar{a}.\,\bar{b}=(\ell_1\ell_2+m_1m_2+n_1n_2)\,,\bar{a}\times\bar{b}=\sqrt{(\ell_1m_2-\ell_2m_1)^2+(m_1n_2-m_2n_1)^2+(n_1\ell_2-n_2\ell_1)^2+^2}$

$(\ell_1m_2-\ell_2m_1)^2+(m_1n_2-m_2n_1)^2+(n_1\ell_2-n_2\ell_1)^2+(\ell_1\ell_2+m_1m_2+n_1n_2)^2$ = $\left|\bar{a}.\,\bar{b}\right|^2+\left|\bar{a}\times\bar{b}\right|^2=|\bar{a}|^2|\bar{b}|^2=({l_1}^2+{m_1}^2+{n_1}^2)({l_2}^2+{m_2}^2+{n_2}^2)=1$

Since $(\ell_1,m_1,n_1),(\ell_2,m_2,n_2)$ are dc's of two lines .

16) If α,β,γ are the roots of $x^3+px^2+qx+r=0$ then$(\alpha+\beta)\,(\beta+\gamma)\,(\gamma+\alpha)=$

A) pq-r B) r-pq C) p + pqr D) pq + r

Correct Answer: B

Solution: We have to find equation whose roots are $\alpha+\beta$,$\gamma+\beta$ and $\alpha+\gamma$.

Assume $y=\alpha+\beta\Rightarrow\alpha+\beta+\gamma-\gamma=-p-\gamma\Rightarrow\gamma=-p-y.$

After substituting $-p-y$ in the place of γ of an equation $(\gamma)^3+p(\gamma)^2+q(\gamma)+r=0$

$(-p-y)^3+p(-p-y)^2+q(-p-y)+r=0$

$-y^3-3p^2y-3py^2-p^3+p^3+y^2p+2p^2y-qp-qy+r=0$

$\Rightarrow y^3+(p^2+q)y+(2p-1)y^2+qp-r=0$

Product of the roots $(\alpha+\beta)\,(\beta+\gamma)\,(\gamma+\alpha)=-(pq-r)$

17) The value of $\frac{\theta}{2}$ in the lagranges mean value theorem for $f(x)=x^3,a=1,h=\frac{1}{2}$ is

A) $\frac{1}{3}$ B) $\sqrt{\frac{19}{56}}$ C) $\sqrt{\frac{19}{24}}-1$ D) $\sqrt{\frac{19}{3}}-2$

Correct Answer: C

Solution: $f(a+h)=f(a)+f'(a+\theta h)$

$f(1+\frac{1}{2})=f(1)+f'(1+\frac{\theta}{2})$

$\frac{19}{8}=3\left(1+\frac{\theta}{2}\right)^2$

$\theta=\sqrt{\frac{19}{24}}-1$

18) If $tanxcoth(\frac{x}{2})=1$ then Cos2xCoshx =

A) 1 B) 2 C) 0 D) -1

Correct Answer: A
Solution: $\tan x \coth\left(\frac{x}{2}\right) = 1$

$\tan x = \tanh\left(\frac{x}{2}\right)$

$\text{Cosh}\, x = \frac{1+\tanh^2\left(\frac{x}{2}\right)}{1-\tanh^2\left(\frac{x}{2}\right)} = \frac{1+\tan^2 x}{1-\tan^2 x} = \sec 2x$

$Hence\ \text{Cos}\, 2x.\, \text{Cosh}\, x = 1$

19) In a triangle ABC $a = 5, b = 4$ and $Tan\frac{C}{2} = \sqrt{\frac{7}{9}}$ then c=

A) 6 B) 3 C) 2 D) 5

Correct Answer: A
Solution: $CosC = \frac{1-Tan^2\frac{C}{2}}{1+Tan^2\frac{C}{2}} = \frac{1-\frac{7}{9}}{1+\frac{7}{9}} = \frac{1}{8}$

$c^2 = a^2 + b^2 - 2ab\cos C = 25 + 16 - 40 \times \frac{1}{8} = 36 \Rightarrow c = 6$

20) If $|z_1| = 2, |z_2 = 3, |z_3| = 4$ and $|z_1 + z_2 + z_3| = 5$ then $|4z_2z_3 + 9z_3z_1 + 16z_1z_2|$ =

A) 20 B) 24 C) 48 D) 120

Correct Answer: D
Solution: $|z_1| = 2, |z_2| = 3, |z_3| = 4 \Rightarrow |z_1|^2 = 4, |z_2|^2 = 9, |z_3|^2 = 16$

$|z_3| = 4 \Rightarrow |z_1|^2 = 4, |z_2|^2 = 9, |z_3|^2 = 16$

$z_1\overline{z_1} = 4, z_2\overline{z_2} = 9, z_3\overline{z_3} = 16$

$\Rightarrow |4z_2z_3 + 9z_3z_1 + 16z_1z_2| = |z_1\overline{z_1}z_2z_3 + \overline{z}_2z_2z_3z_1 + \overline{z}_3z_3z_1z_2|$

$= |z_1||z_2||z_3||z_1 + z_2 + z_3| = 120$ since $\overline{|z_1 + z_2 + z_3|} = |z_1 + z_2 + z_3|$

21) If $1, \alpha_1, \alpha_2 \ldots\ldots \alpha_{n-1}$ are the nth roots of unity. The value of $(3-\alpha)\left(3-\alpha^2\right)\ldots.\left(3-\alpha^{n-1}\right)$ is

A) n B) 0 C) $\frac{(3^n-1)}{2}$ D) $\frac{(3^n+1)}{2}$

Correct Answer: C
Solution: $(x^n - 1) = (x-1)(x-\alpha)(x-\alpha^2)\ldots\ldots\ldots(x-\alpha^{n-1})$ Substitute x= 3 weget $(3-\alpha)\left(3-\alpha^2\right)\ldots.\left(3-\alpha^{n-1}\right) = \frac{3^n-1}{2}$.

22) One value of $(1+i)^{\frac{1}{2}}\ is\ 2^{\frac{1}{4}}e^{\frac{i\pi}{8}}$ .Then other value is

A) $2^{1/4}e^{-i(\pi/8)}$ B) $2^{1/4}e^{i(5\pi/8)}$ C) $2^{1/4}e^{-i(5\pi/8)}$ D) $2^{1/4}.e^{+i(9\pi/8)}$

Correct Answer: D

Solution: $(1+i)^{\frac{1}{2}} = \left(\sqrt{2}\right)^{\frac{1}{2}}\left(\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}\right)^{\frac{1}{2}} = 2^{\frac{1}{4}}\left(\text{Cos}\,\frac{\pi}{4}+i\,\text{Sin}\,\frac{\pi}{4}\right)^{\frac{1}{2}}$

$= 2^{\frac{1}{4}}\left(\text{Cos}\left(2k\pi+\frac{\pi}{4}\right)+i\,\text{Sin}\left(2k\pi+\frac{\pi}{4}\right)\right)^{\frac{1}{2}}$

$= 2^{\frac{1}{4}}\left(\text{Cos}\left(k\pi+\frac{\pi}{8}\right)+i\,\text{Sin}\left(k\pi+\frac{\pi}{8}\right)\right)\ \ where\ \ k=0\,,1.$

One root is $2^{\frac{1}{4}}e^{\frac{i\pi}{8}}$ Other root is $2^{\frac{1}{4}}e^{\frac{i9\pi}{8}}$

23) If the ratio of the roots of $ax^2+bx+c=0$ is m : then $\frac{b^2}{ac}=$

A) $\frac{(m+n)^2}{mn}$ B) $\frac{m+n}{mn}$ C) $\frac{mn}{(m+n)^2}$ D) $\frac{mn}{(m+n)}$

Correct Answer: A

Solution: Let roots of the equation be in the ratio be m : n

then roots are $m\alpha, n\alpha$

Sum of the roots are $m\alpha+n\alpha=\frac{-b}{a}, mn(\alpha)^2=\frac{c}{a}$

$\frac{(m+n)^2}{mn}=\frac{\frac{b^2}{a^2}}{\frac{c}{a}}=\frac{b^2}{ac}$

24) If the roots of $4x^3-12x^2+11x+k=0$ are in A.P. then K =

A)-3 B)1 C)2 D)3

Correct Answer: A

Solution: Let roots of equation be $a-d, a, a+d.$

Sum of the roots $3a=3\Rightarrow a=1$

$4\times 1-12\times 1+11\times 1+k=0\Rightarrow k=-3$

25) If $\left(x^2-x+1\right)$ is a factor of $f(x)=ax^3+bx^2+cx+d$ where and a,b,c,d are real, then the real root of $f(x)=0$ is

A) $\frac{a}{d}$ B) $\frac{d}{a}$ C) $\frac{-a}{d}$ D) $\frac{-d}{a}$

Correct Answer: D

Solution: Roots of are $x^2-x+1=0$ are $-w, -w^2.$

Let the roots of f(x) = 0 be $-w-w^2, \alpha.$

Product of the roots = $(-w)\left(-w^2\right)\alpha=-\frac{d}{a}\therefore\alpha=-d/a$

26) If $-2, 5, 7, -11$ are the roots of $ax^4 + bx^3 + cx^2 + dx + e = 0$ then the roots of $ax^4 - bx^3 + cx^2 - dx + e = 0$ are

A) 2,5,7,11 B) 2,-5,-7,11 C) 2,5,-7,-11 D) -2,5,7,11

Correct Answer: B
Solution: If α is a toot of $f(x) = 0$ then $-\alpha$ is a root of $f(-x) = 0$.If $-2, 5, 7, -11$ are the roots of $ax^4 + bx^3 + cx^2 + dx + e = 0$ then roots of equation $ax^4 - bx^3 + cx^2 - dx + e = 0$ are $2, -5, -7, 11$

27) The equation of the image of the circle $x^2 + y^2 - 6x - 4y + 12 = 0$ by the line mirror $x + y - 1 = 0$.

A) $x^2 + y^2 + 2x + 4y + 4 = 0$ B) $x^2 + y^2 - 2x + 4y + 4 = 0$

C) $x^2 + y^2 + 2x + 4y - 4 = 0$ D) $x^2 + y^2 + 2x - 4y + 4 = 0$

Correct Answer: A
Solution: Image of the centre $(3, 2)$ of the circle with repsect to the line $x + y - 1 = 0$,we get $(3, 2)$.
$\frac{x-3}{1} = \frac{y-2}{1} = \frac{-2(3+2-1)}{2} = -4$

$\Rightarrow x = -1, y = -2 \Rightarrow$ Centre =$(1, 2)$ and radius = 1.

Hence equation of circle is $x^2 + y^2 + 2x + 4y + 4 = 0$

28) The equation of a circle with centre $(4, 1)$ and having $3x + 4y - 1 = 0$ as tangent is

A) $x^2 + y^2 - 8x - 2y - 8 = 0$ B) $x^2 + y^2 - 8x - 2y + 8 = 0$

C) $x^2 + y^2 - 8x + 2y + 8 = 0$ D) $x^2 + y^2 - 8x - 2y + 4 = 0$

Correct Answer: B
Solution: Perpendicular distance from centre to the line $\frac{12+4-1}{\sqrt{3^2+4^2}} = 3$= Radius.

Hence equation of circle $(x-4)^2 + (y-1)^2 = 9 = (x-4)^2 + (y-1)^2 = 9$

29) The sum of all four digited numbers that can be formed using the digits 0, 2, 4, 7, 8 without repetition is

A) 479952 B) 497952 C) 545958 D) 547598

Correct Answer: C
Solution: Value carrried by 2 in the units place $= \left({}^4P_3 - {}^3P_2\right) 2$

By putting $'2'$ in the units place, remaining 3 places can be filled by using digits $0, 4, 7, 8$ in ${}^4P_3 - {}^3P_2$ ways.Where ${}^3P_2$ is the umber of four digited numbers which start with zero and which containing $'2'$ in the units place.

Value carried by 2 present in the tens place = $\left({}^4P_3 - {}^3P_2\right) 20$

Value carried by 2 present in the hundreds place = $\left({}^4P_3 - {}^3P_2\right) 200$

Value carried by 2 present in the thousands place = $\left({}^4P_3\right) 2000.$

Value carried by '2' = $({}^4P_3)2000 + ({}^4P_3 - {}^3P_2)222,$

Simalrly value carried by the remaining digits

$({}^4P_3)4000 + ({}^4P_3 - {}^3P_2)444,$

$({}^4P_3)7000 + ({}^4P_3 - {}^3P_2)777$

$({}^4P_3)8000 + ({}^4P_3 - {}^3P_2)888$

Value carried by the digits $2, 4, 7, 8$ is equal to
$({}^4P_3)(8000 + 7000 + 4000 + 2000) + ({}^4P_3 - {}^3P_2)(888 + 777 + 222 + 444) = 545958$

30) The sum of the digits at the ten's place of all the numbers formed with the help of 3, 4, 5, 6 taken all at a time is

A) 432 B) 108 C) 136 D) 180

Correct Answer: B
Solution: Value carried by '3' in the tens place is equal to 30.Number of four digited numbers which contains '3' in the tens place = 3!.Hence valuue carried by '3' $= 6 \times 30 = 180$

Similarly value carried by 4 , 5 , 6 are repectively $240, 300, 360.$ Hence value carried by all digits which are present in the tens place $= 1080$

sum of the digits $= \frac{1080}{10} = 108$

31) A and B are two independent events. The probability that both A and B occur is $\frac{1}{6}$ and the probability that neither of them occur is $\frac{1}{3}$. The probability of occurence of A is

A) $\frac{1}{4}, \frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{3}, \frac{1}{2}$ D) $\frac{1}{4}$

Correct Answer: C
Solution: The probability that both A and B occur is $P(A \cap B) = \frac{1}{6}$

probability that neither of them occur is = $P(\bar{A} \cap \bar{B}) = \frac{1}{3}$

Let $P(A) = x, P(B) = y.$ Equations becomes $xy = \frac{1}{6}$ and

$(1 - x)(1 - y) = \frac{1}{3}$.Solving equations we get $x = \frac{1}{3}\, y = \frac{1}{2}$

32) A and B alternately cut a card each from a pack of cards with replacement and pack is shuffled after each cut. If A starts the game and the game is continued till one cuts a spade, the respective probabilities of A and B cutting a spade are

A) $\frac{1}{3}, \frac{2}{3}$ B) $\frac{3}{4}, \frac{1}{4}$ C) $\frac{4}{7}, \frac{3}{7}$ D) $\frac{3}{7}, \frac{4}{7}$

Correct Answer: C
Solution: Probablity of selecting a spade $p = \frac{13}{52} = \frac{1}{4}$

Probability of not selecting a spade $q = 1 - \frac{1}{4} = \frac{3}{4}$,

Probability that A cuts the spade $= p + q^2p + q^4p + \ldots\ldots\ldots\ldots\ldots = p(A) = \frac{p}{1-q^2}$

$p(A) = \frac{p}{1-q^2} = \frac{4}{7}$

$p(B) = 1 - \frac{4}{7} = \frac{3}{7}$

33) If $I_n = \int \frac{\sin nx}{\sin x}\, dx \ (n > 1)$ then $I_{n+1} - I_{n-1}$

A) $\frac{2}{n-1}\cos(n-1)x$ B) $\frac{2}{n-1}\sin(n-1)x$ C) $\frac{2}{n}\cos nx$ D) $\frac{2}{n}\sin nx$

Correct Answer: D

Solution: $\sin(n+1)x - \sin(n-1)x = 2\cos nx \sin x$

$I_{n+1} - I_{n-1} = \int \frac{\sin(n+1)x - \mathrm{Sin}(n-1)x}{\sin x}\, dx$

For $n > 1$

$= \int 2\cos nx dx$ = $\frac{2\,\mathrm{Sin}\,nx}{n} + c$

34) $\int_0^{50} (x - [x])dx$

A) 25 B) 20 C) 15 D) 10

Correct Answer: A

Solution: If 'f ' is peroidic function with peroid 'a' $\int_0^{na} f(x)dx = n\int_0^a f(x)dx$.

$x - [x]$ is peroidic function with period $'1'$

$\int_0^{50} (x - [x])dx = 50\int_0^1 (x - [x])dx$

$0 < x < 1 \Rightarrow [x] = 0 \Rightarrow 50\int_0^1 (x - [x])dx$ = 25.

35) $\int_8^9 \sqrt{\frac{x-8}{9-x}}\, dx$

A) π B) $\frac{\pi}{3}$ C) $\frac{\pi}{2}$ D) $\frac{\pi}{6\sqrt{2}}$

Correct Answer: C

Solution: $\int_a^b \sqrt{\frac{x-a}{b-x}}dx = \frac{\pi}{2}(b-a) \Rightarrow \int_8^9 \sqrt{\frac{x-8}{9-x}}\, dx = \frac{\pi(9-8)}{2} \Rightarrow \frac{\pi}{2}$

36) $\int_1^5 \frac{1}{\sqrt{(x-1)(5-x)}}\,dx$

A) $\frac{\pi}{2}$ B) π C) $\frac{\pi}{4}$ D) $\frac{-\pi}{2}$

Correct Answer: B

Solution: $\int_a^b \frac{1}{\sqrt{(x-a)(b-x)}}\,dx = \pi$ for all 'a' and 'b'.

37) Area of the region bounded by $y = |x|\ and\, y = 1 - |x|$ is

A) $\frac{1}{3}$ B) 1 C) $\frac{1}{2}$ D) 2

Correct Answer: C

Solution: $y = |x|\ and\, y = 1 - |x|$ intersect at $x = \frac{1}{2}, x = \frac{-1}{2}$.

Points of intersection $(\frac{1}{2}, \frac{1}{2}), (\frac{-1}{2}, \frac{1}{2})$.

It forms a square of side $\frac{1}{\sqrt{2}}$.

Area of a square $= \frac{1}{2}$.

or $A = 2\int_0^1 (1 - x - x)dx = \frac{1}{2}$

38) The Differential equation of all non-vertical lines in a plane is

A) $\frac{d^2y}{dx^2} = 0$ B) $\frac{d^2x}{dy^2} = 0$ C) $\frac{dy}{dx} = 0$ D) $\frac{dx}{dy} = 0$

Correct Answer: A

Solution: Equation of a line is $ax + by + c = 0$

Equation of a vertical line is $x = a$ i.e possible if $b = 0$ i.e

Non vertical lines is possible if $b \neq 0$

$by = -ax - c \Rightarrow y = -\frac{a}{b}x - \frac{c}{b} \Rightarrow y = lx + m$

Differentiaing with respect to x we get $y' = l \Rightarrow y'' = 0$

39) The order, degree of the D.E whose solution is ,$Ax^2 + By^2 = 1$ where A, B are arbitrary constants , are

A) 1,2 B) 1,1 C) 2,1 D) 2,2

Correct Answer: C

Solution: Equation of the curve $Ax^2 + By^2 = 1$

Differentiating with respect to x we get $A(2x) + B(2yy') = 0 \Rightarrow Ax + By' = 0.....(1)$

Differentiating again we get $A+B(yy''+(y')^2)=0.....(2)$

Solving (1) and (2) we get

$\frac{A}{B}+\frac{y'}{x}=0, \frac{A}{B}+(yy''+(y')^2)=0.....(2)\Rightarrow -\frac{y'}{x}+(yy''+(y')^2)=0$

It contains second order derivative with degree 1 .

Hence order of differential equation is 2 and degree is 1.

40) $Cos^{-1}\left\{\frac{1}{\sqrt{2}}\left(Cos\frac{9\pi}{10}-Sin\frac{9\pi}{10}\right)\right\}=$

A) $\frac{3\pi}{20}$ B) $\frac{7\pi}{10}$ C) $\frac{7\pi}{20}$ D) $\frac{17\pi}{20}$

Correct Answer: D

Solution: $Cos^{-1}\left\{\frac{1}{\sqrt{2}}\left(Cos\frac{9\pi}{10}-Sin\frac{9\pi}{10}\right)\right\}=$

$Cos^{-1}\left\{\left(\text{Cos}\frac{\pi}{4}Cos\frac{9\pi}{10}-\text{Sin}\frac{\pi}{4}Sin\frac{9\pi}{10}\right)\right\}=$

$Cos^{-1}\left\{\left(\text{Cos}\left(\frac{\pi}{4}+\frac{9\pi}{10}\right)\right)\right\}=$

$Cos^{-1}\left\{\left(\text{Cos}\left(\frac{23\pi}{20}\right)\right)\right\}=$

$Cos^{-1}\left\{\left(\text{Cos}\left(2\pi-\frac{17\pi}{20}\right)\right)\right\}=$

$Cos^{-1}\left\{\left(\text{Cos}\left(\frac{17\pi}{20}\right)\right)\right\}=$

$=\frac{17\pi}{20}$

41) AB is a focal chord of the parabola . If $A=(4a,4a)$ then B =

A) $(\frac{a}{2},\frac{-a}{4})$ B) $(\frac{a}{4},-a)$ C) $(\frac{a}{2},\frac{-a}{2})$ D) $(\frac{a}{4},-4a)$

Correct Answer: B

Solution: One end of focal chord of a parabola $y^2=4ax$ is $(at^2,2at)$ then other end of it is $(\frac{a}{t^2},-\frac{2a}{t})$.since $at^2=4a\Rightarrow t=2$.Hence other end of the focal chord is $(\frac{a}{4},-a)$

42) If $(-4,1),(6,1)$are the vertices of an Ellipse and one of the foci lies on x-2y = 2 then the eccentricity is

A) $\frac{3}{5}$ B) $\frac{4}{5}$ C) $\frac{2}{5}$ D) $\frac{1}{5}$

Correct Answer: A

Solution: Centre of the ellipse $\left(\frac{-4+6}{2},\frac{1+1}{2}\right)=(1,1).$

Focii $(X,Y)=(\pm ae,0)$

$\Rightarrow (x-1,y-1)=(\pm ae,0)\Rightarrow (x,y)=(1\pm ae,1).$

Hence in one case $(1+ae,1)$ lies on the line $x-2y=2$

$\Rightarrow 1+ae-2=2\Rightarrow ae=3, 2a=10\Rightarrow 3e=4\Rightarrow e=3/5$

43) If angle between the asymptotes of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is 45^o , then value of ecentricity e is

A) $\sqrt{4\pm2\sqrt{2}}$ B) $\sqrt{4+2\sqrt{2}}$ C) $\sqrt{4-2\sqrt{2}}$ D) $\sqrt{4-3\sqrt{2}}$

Correct Answer: C

Solution: Angle between asymptotes = $2\tan^{-1}\left(\frac{b}{a}\right)=45^o \Rightarrow \frac{b}{a}=22\frac{1}{2}^o$

$\Rightarrow \frac{a^2(e^2-1)}{a^2}=\left(\sqrt{2}-1\right)^2$

$\frac{a^2(e^2-1)}{a^2}=\left(\sqrt{2}-1\right)^2$

$e=\sqrt{4-2\sqrt{2}}$

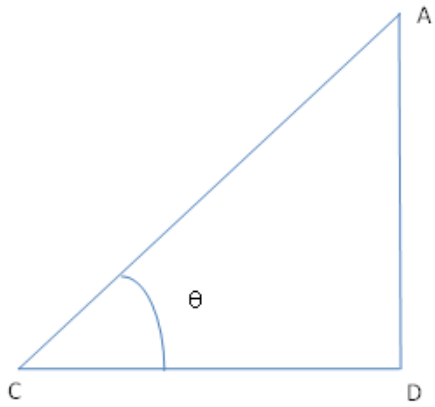
44) A tree is broken by wind, its upper part touches the ground at a point 10 meters from the foot of the tree and makes an angle of 45^o with the ground. The entire length of the tree is

A) 15 mt B) 20 mt C) $10(1+\sqrt{2})$ mt D) $10\left(1+\frac{\sqrt{3}}{2}\right)$ mt

Correct Answer: C

Solution: Let 'h' be the height of the tree. Let length of the broken tree be 'm'.

Broken part makes an angle 45^o with the ground



CD = 10m

$Tan45^o=\frac{h}{10}\Rightarrow h=10$

$Sin45^o=\frac{h}{m}=\frac{1}{\sqrt{2}}\Rightarrow m=\sqrt{2}h$

Length of the tree $=h+\sqrt{2}h=10(1+\sqrt{2})$

45) $\lim\limits_{x\to0}\left[\frac{16^x+9^x}{2}\right]^{\frac{1}{x}}=$

A) 1 B) 12 C) $\frac{25}{2}$ D) 144

Correct Answer: C

Solution: $\lim\limits_{x\to 0}\left[\frac{16^x+9^x}{2}\right]^{\frac{1}{x}}$ is of the form $(1)^\infty$

which can be evaluated with formula $\underset{x\to a}{lt}\, f(x)^{g(x)} = e^{\underset{x\to a}{lt}\, g(x)(f(x-1)}$

$$\underset{x\to 0}{lt}\, e^{\frac{1}{x}\left(\frac{16^x+9^x-2}{2}\right)} = \underset{x\to 0}{lt}\, e^{\left(\frac{16^x+9^x-2}{2x}\right)} = \underset{x\to 0}{lt}\, e^{\left(\frac{16^x\log 16+9^x\log 9}{2}\right)}$$

$$= \underset{x\to 0}{lt}\, e^{\left(\frac{\log 16+\log 9}{2}\right)} = \underset{x\to 0}{lt}\, e^{log\frac{25}{2}} = \frac{25}{2}$$

46) $\lim\limits_{x\to 0}\left[\frac{\sin(\text{sgn}(x))}{(\text{sgn}(x))}\right]$ where [ ]denotes the greatest integer function, is equal to

A) 0 B) 1 C) -1 D) Does not exist

Correct Answer: B

Solution: If $x>0$ then $sgnx=1, sgn(sgn(x)=sgn(1)=1$ since $1>0$

As $x\to 0^+$

$\lim\limits_{x\to 0}\left[\frac{\sin(\text{sgn}(x))}{(\text{sgn}(x))}\right]=\lim\limits_{x\to 0}\frac{1}{1}=1$

If $x\to 0^-\Rightarrow sgn(x)=-1\Rightarrow sgn(-1)=-1$ since $-1<0$

$\lim\limits_{x\to 0}\left[\frac{\sin(\text{sgn}(x))}{(\text{sgn}(x))}\right]=\lim\limits_{x\to 0}\frac{-1}{-1}=1.$

Hence limit exists and equals to 1.

47) If $\lim\limits_{n\to\infty}\frac{n.3^n}{n(x-2)^n+n.3^{n+1}-3^n}=\frac{1}{3}, n\in N$ then the range of x' is

A) $[2,5)$ B) $(1,5)$ C) $(-1,5)$ D) $(-\infty,\infty)$

Correct Answer: B

Solution: $\lim\limits_{n\to\infty}\frac{n.3^n}{n(x-2)^n+n.3^{n+1}-3^n}=\frac{1}{3}$

$\lim\limits_{n\to\infty}\frac{1}{\frac{(x-2)^n}{3^n}+3-\frac{1}{n}}=\frac{1}{3}$

Dividing numerator and denominator $n\times 3^n$

For $\lim\limits_{n\to\infty}$ to be equal to $\frac{1}{3}$

$\lim\limits_{n\to\infty}\frac{1}{n}\to 0$ and $\lim\limits_{n\to\infty}\left(\frac{x-2}{3}\right)^n\to 0$

Which is possible if $\frac{|x-2|}{3}|<1$

i.e $-3+2<x<3+2\Rightarrow x\in(-1,5)$

48) $y=\sin(\sin x)\Rightarrow y_2+y_1 tanx=$

A)$-y\cos^2 x$ B)$y\cos^2 x$ C)$-y\sin^2 x$ D)$y\sin^2 x$

Correct Answer: A
Solution: $y_1 = \cos(\sin x)\cos x$

$y_2 = -\sin(\sin x)\cos^2 x + \cos(\sin x)(-\sin x)$

$y_2 = -y\cos^2 x + \frac{y_1}{\cos x}(-\sin x)$

$\Rightarrow y_2 + y_1 tanx = -y\cos^2 x$

49) If $x = a\cos^4 t, y = b\sin^4 t$ then $\frac{dy}{dx}$ at $t = \frac{3\pi}{4}$ is

A)$-\frac{b}{a}$ B)$\frac{b}{a}$ C)$-\frac{a}{b}$ D) $\frac{a}{b}$

Correct Answer: A
Solution: $\frac{dy}{dx} = \frac{4b\sin^3 t\ \cos t}{-4a\cos^3 t\ \sin\ t} = \frac{-b}{a}\tan^2 t$

$= (\frac{dy}{dx})_{t=\frac{3\pi}{4}} = \frac{-b}{a}$

50) $f(x) = (1+x)^{\frac{5}{x}},\ x \neq 0, f(0) = e^5$ at $x = 0$, f is

A) continuous B) discontinuous C) not determined D) none

Correct Answer: A
Solution: $\underset{x\to 0}{Lt} f(x) = \underset{x\to 0}{Lt}(1+x)^{\frac{5}{x}} = \underset{x\to 0}{Lt}((1+x)^{\frac{1}{x}})^5 = e^5$

$f(0) = e^5 \Rightarrow \underset{x\to 0}{Lt} = f(0)$

Hence f is continuous at $x = 0$

51) If the mean of a Binomial Distribution is 25, then standard deviation lies in the interval

A) $[0,\ 5)$ B) $[0,\ 25)$ C) $(0,\ 5]$ D) $(0,\ 5)$

Correct Answer: A
Solution: If mean $= \mu = np \Rightarrow S.D = \sqrt{npq}$

If $q = 1$ then $p = 0 \Rightarrow np = 0$ which is not possible.

Hence $0 \leqslant q < 1 \Rightarrow 0 \leqslant npq < np \Rightarrow\ 0 \leqslant \sqrt{npq} < \sqrt{np}$

$\Rightarrow S.D \in \left[0, \sqrt{\mu}\right) = \left[0, \sqrt{25}\right) = [0, 5).$

52) If $Cos(\frac{x}{2})Cos(\frac{x}{2^2})\cos(\frac{x}{2^3})\ ...\infty = \frac{\text{sinx}}{\text{x}}$ then
$\frac{1}{2}\tan\frac{x}{2} + \frac{1}{2^2}\tan\left(\frac{x}{2^2}\right) + \frac{1}{2^3}\tan\left(\frac{x}{2^3}\right) + - - -\infty =$ _____

A) $\frac{x\cot x-1}{x}$ B) $\cot x$ C) $\frac{x\tan x-1}{x}$ D) $\frac{1-x\cot x}{x}$

Correct Answer: D

Solution: $\cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{2^2}\right)\cos\left(\frac{x}{2^3}\right)\ldots\infty=\frac{\sin x}{x}$

Taking logarithms both sides we get

$logcos\frac{x}{2}+logcos\frac{x}{2^2}+logcos\frac{x}{2^2}+\ldots\ldots\ldots=logsinx-logx$

Differentiating both sides with respect to x we get

$\frac{-\sin(x/2)}{\cos(x/2)}\frac{1}{2}-\frac{\sin(x/2^2)}{\cos(x/2^2)}\frac{1}{4}-\ldots\ldots\ldots\ldots\ldots=\frac{\cos x}{\sin x}-\frac{1}{x}$

$\Rightarrow\frac{1}{2}\tan\frac{x}{2}+\frac{1}{2^2}\tan\left(\frac{x}{2^2}\right)+\frac{1}{2^3}\tan\left(\frac{x}{2^3}\right)+---\infty=\_\_\_\_\_=\frac{1}{x}-cotx$

53) $A=\begin{pmatrix}n & 1\\ 1 & 0\end{pmatrix}$ and A^2 is an identity then $n=$

A) 1 B) -1 C) 0 D) 2

Correct Answer: C

Solution: $A^2=\begin{pmatrix}n & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}n & 1\\ 1 & 0\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

By comparing elements in the first position after simplification

$n^2+1=1\Rightarrow n=0$

54) If $\Delta=\begin{vmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix}=5$

then the value of $\begin{vmatrix}b_2c_3-b_3c_2 & a_3c_2-a_2c_3 & a_2b_3-a_3b_2\\ b_3c_1-b_1c_3 & a_1c_3-a_3c_1 & a_3b_1-a_1b_3\\ b_1c_2-b_2c_1 & a_2c_1-a_1c_2 & a_1b_2-a_2b_1\end{vmatrix}$

A) 5 B) 10 C) 20 D) 25

Correct Answer: A

Solution: Each element in the required detrminant is a cofactor of elements of a matix Δ

i.e we have to find determinant of adjoint of a matrix

$det(adjA)=|A|^2=25$

55) The angle between the asymptotes of the hyperbola $3x^2-y^2=3$ is

A) $\frac{\pi}{6}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{3}$ D) $\frac{2\pi}{3}$

Correct Answer: D

Solution: Equation of hyperbola $3x^2 - y^2 = 3 \Rightarrow \frac{x^2}{1} - \frac{y^2}{3} = 1$

Angle betweeen assymptotes $= 2tan^{-1}\frac{b}{a} = 2tan^{-1}\frac{\sqrt{3}}{1} = \frac{2\pi}{3}$

56) A fair die is rolled 180 times. The expected number of six's is

A) 50 B) 30 C) 10 D) 5

Correct Answer: B

Solution: If a fair die is thrown then probability of getting a number 6 is $= \frac{1}{6}$

Expected number of six's if die is thrown 180 times is $180 \times \frac{1}{6} = 30$

57) If $f(x) = x^2 + x + 1$ where$x = 2$, $\delta x = 0.01$ then $\delta f =$

A) 0.0501 B) 0.05 C) 0.005 D) 0.0051

Correct Answer: A

Solution:

$\delta f = f(x + \delta x) - f(x) = f(2 + 0.01) - f(2) = f(2.01) - f(2) = (2.01)^2 + 2.01 + 1 - (2^2 + 2 + 1)$

$= 0.0401 + 0.01 = 0.0501$

58) The first negative term in the expansion of $(1 + x)^{\frac{3}{4}}$ is if x is positive.

A) T_2 B) T_4 C) T_3 D) T_7

Correct Answer: C

Solution: $T_{r+1} = \frac{n(n-1)(n-2)(n-3).....(n-(r-1))}{r!}x^r$

First negative term occurs if $n - r + 1 < 0 \Rightarrow \frac{3}{4} - r + 1 < 0 \Rightarrow r > \frac{7}{4} = 1.75$

i.e $r = 2 \Rightarrow Term = T_{r+1} = T_3$

59) A line segment of 2 units is sliding with its ends on two perpendicular lines. Then the locus of the middle point is

A) $x + 2y + 1 = 0$ B) $x^2 + y^2 = 1$ C) $y^2 = 4ax$ D) $x^2 = 4ay$

Correct Answer: B

Solution: Let equation of line AB intercepted between coordinated axes is $\frac{x}{a} + \frac{y}{b} = 1$.

A(a, 0), B(0,b)

Let (x_1, y_1) be the midpoint of the line which is intercepted between the perpendicular lines.

$\Rightarrow (\frac{a+o}{2}, \frac{0+b}{2}) = (x_1, y_1) \Rightarrow a = 2x_1, b = 2y_1$

$AB = 2 \Rightarrow \sqrt{a^2 + b^2} = 2 \Rightarrow 4({x_1}^2 + {y_1}^2) = 4$

$\Rightarrow$Locus of midpoint is $x^2 + y^2 = 1$

60) The number of partial fractions of $\frac{x^3-3x^2+3x}{(x-1)^5}$ is

A) 2 B) 3 C) 4 D) 5

Correct Answer: A

Solution: $\frac{x^3-3x^2+3x}{(x-1)^5}$

Let $x - 1 = y \Rightarrow \frac{x^3-3x^2+3x}{(x-1)^5}$

$= \frac{(y+1)^3-3(y+1)^2+3(y+1)}{y^5}$

$= \frac{y^3+3y^2+3y+1-3y^2-6y-3+3y+3}{y^5}$

$= \frac{y^3+1}{y^5} = \frac{1}{y^2} + \frac{1}{y^5}$

Hence number of partial fractions are equal to 2.

61) The number of partial fractions of $\frac{2}{x^4+x^2+1}$ is

A) 2 B) 3 C) 4 D) 5

Correct Answer: A

Solution: $\frac{2}{x^4+x^2+1} = \frac{2}{x^4+2x^2+1-x^2} = \frac{2}{(x^2+1)^2-x^2} = \frac{Ax+B}{x^2+x+1} + \frac{Cx+D}{x^2-x+1}$

Hence number of partial fractions are = 2

62) $C_0 + \frac{C_1}{2}x + \frac{C_2}{3}x^2 + \ldots\ldots + \frac{C_n}{n+1}x^n =$

A) $\frac{(1+x)^{n+1}-1}{(n+1)x}$ B) $\frac{(1+x)^n-1}{(n+1)x}$ C) $\frac{(1+x)^{n+1}+1}{(n+1)x}$ D) $\frac{(1+x)^{n+2}-1}{(n+1)x}$

Correct Answer: A

Solution: $(1 + x)^n = C_0 + C_1x + C_2x^2 + C_3x^3 + \ldots\ldots\ldots\ldots C_nx^n$

Integrating both sides with respect to x we get

$C_0x + C_1\frac{x^2}{2} + \ldots\ldots C_n\frac{x^n}{n+1} = \frac{(1+x)^{n+1}}{(n+1)} + k$

Substituting $x = 0$ we get

$0 = \frac{1}{n+1} + k \Rightarrow k = -\frac{1}{n+1}$

$\Rightarrow C_0 + \frac{C_1}{2}x + \frac{C_2}{3}x^2 + \ldots\ldots + \frac{C_n}{n+1}x^n = \frac{(1+x)^{n+1}-1}{(n+1)x}$

63) $\int_{\pi}^{10\pi} |\sin x|\ dx\ =$

A) 20 B) 8 C) 10 D) 18

Correct Answer: D

Solution: $\int\limits_{mT}^{nT} f\left(x\right)\ dx = (n-m)\int\limits_{0}^{T} f\left(x\right)\ dx$ where T is the peroid of the function $f(x)$

$f(x) = |Sinx|$ whose peroid is π

$\int_{\pi}^{10\pi} |\sin x|\ dx\ = 9\int\limits_{0}^{\pi} f\left(x\right)\ dx = 9\int\limits_{0}^{\pi} |sinx|dx = 9\int\limits_{0}^{\pi} sinxdx = 18$

64) The solution of $\left(x+y+1\right)\frac{dy}{dx} = 1$ is

A) $x = -\left(y+2\right) + ce^{y}$ B) $y = -\left(x+2\right) + ce^{x}$ C) $x = -\left(y+2\right) + ce^{x}$

D) $x = \left(y+2\right) + ce^{-y}$

Correct Answer: A

Solution: $\frac{dx}{dy} = x + y + 1$

$xe^{-y} = -(y+2) + ce^{y}$

65) Sum to '10' terms of series $1 + (1+3) + (1+3+5) + \ldots\ldots\ldots$

A) 385 B) 425 C) 445 D) 625

Correct Answer: A

Solution: nth term of the series $1 + (1+3) + (1+3+5) + \ldots\ldots\ldots = 1+3+5+\ldots.(2n-1)$

$t_n = 1+3+5+\ldots.(2n-1) = n^2$

$S_n = \frac{n(n+1)(2n+1)}{6}$

$S_{10} = \frac{10\times11\times21}{6} = 385$

66) If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ and $x \neq y\ then\ \frac{dy}{dx} =$

A) $\frac{1}{1+x}$ B) $\frac{1}{(1+x)^2}$ C) $\frac{-1}{(1+x)^2}$ D) $\frac{1-x}{(1+x)^2}$

Correct Answer: C

Solution: $x\sqrt{1+y} = -y\sqrt{1+x}$

$x^2(1+y) = y^2(1+x)$

$x^2 - y^2 = -x^2y + y^2x$

$(x-y)(x+y) = -xy(x-y) \Rightarrow (x-y)(x+y+xy) = 0$

$x \neq y$ then $x+y+xy = 0 \Rightarrow y = \frac{-x}{x+1} = -1 + \frac{1}{x+1} \Rightarrow \frac{dy}{dx} = \frac{-1}{(x+1)^2}$

67) Maximum value of $y = 1 + 4\sin^2 x\ \cos^2 x$

A) 1 B) 2 C) 3 D) 4

Correct Answer: B

Solution: $y = 1 + 4\sin^2 x\ \cos^2 x = 1 + Sin^2(2x)$

$0 \leq sin^2(2x) \leq 1 \Rightarrow 1 \leq 1 + sin^2(2x) \leq 2$

Maximum value of $1 + 4\sin^2 x\ \cos^2 x = 2$

68) The maximum value of $\cos^3 A + \cos^3(120^0 - A) + \cos^3(120^0 + A)$ is

A) 0.75 B) 0.25 C) 1.5 D) 2.5

Correct Answer: A

Solution:

$CosA + Cos(120^o - A) + Cos(120^o + A) = cosA + 2cos120cosA = cosA + 2 \times \frac{-CosA}{2} = 0$

$a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc$

$\cos^3 A + \cos^3(120^0 - A) + \cos^3(120^0 + A) = 3CosACos(120 - A)Cos(120 + A) = \frac{3}{4}Cos3A$

Maximum value of given function $= \frac{3}{4}$

69) 5,7 are the intercepts of plane on the y-axis, z-axis respectively. If the plane is parallel to the x-axis then the equation of the plane is

A) 5y + 7z = 35 B) 7y + 5z = 1 C) $\frac{y}{7} + \frac{z}{5} = 35$ D) 7y + 5z = 35

Correct Answer: D

Solution: Equation of a plane parallel to x -axis is $ly + mz = k$

y -intercept = 5, z-intercept = 7

$\frac{y}{5} + \frac{z}{7} = 1$

$7y + 5z = 35$

70) The equation of the plane through the origin and containing the lines whose direction cosines are proportional to $(1, -2, 2)$ and $(2, 3, -1)$ is

A) $x - 2y + 2z = 0$ B) $2x + 3y - z = 0$ C) $x + 5y - 3z = 0$ D) $4x - 5y - 7z = 0$

Correct Answer: D

Solution: Equation of plane passing through (x_1, y_1, z_1) and containing the lines whose drs are

proportional to $(a_1, b_1, c_1), (a_2, b_2, c_2)$ is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

Equation of plane passing through origin and containing the lines whose drs are proportional to $(1, -2, 2), (2, 3, 1)$ is

$$\begin{vmatrix} x & y & z \\ 1 & -2 & 2 \\ 2 & 3 & -1 \end{vmatrix} = 0$$

$\Rightarrow 4x - 5y - 7z = 0$

71) $f(x) = \cos(x^2 - 2[x])$, for $0 < x < 1$, $f^1\left(\frac{\sqrt{\pi}}{2}\right)$= ----------

A) $-\sqrt{\frac{\pi}{2}}$ B) $-\sqrt{\pi}$ C) $\sqrt{\frac{\pi}{2}}$ D) $\sqrt{\pi}$

Correct Answer: A

Solution: If $0 < x < 1$ then $[x] = 0 \Rightarrow f(x) = Cosx^2 \Rightarrow f'(x) = -(Sinx^2)(2x)$

$f'(\frac{\sqrt{\pi}}{2}) = -(sin\frac{\pi}{4})(2 \times \frac{\sqrt{\pi}}{2}) = -\frac{\sqrt{\pi}}{\sqrt{2}}$

72) The area enclosed by the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

A) πab B) $\frac{\pi ab}{4}$ C) 4ab D) 8ab

Correct Answer: A

Solution: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\Rightarrow y^2 = b^2(1 - \frac{x^2}{a^2})$

$\Rightarrow y = \pm b\sqrt{1 - \frac{x^2}{a^2}} \Rightarrow \pm\frac{b}{a}\sqrt{a^2 - x^2}$

Replacing x, y with $-x, -y$ we getsame equation .Hene curve is symmetric with respect to both x-axis and y-axis.

Area bounded by ellipse $= \frac{4b}{a}\int_0^a \sqrt{a^2 - x^2}dx = 4\times$ Area of the quadrant of a circle with radius a

$= \frac{4b}{a} \times \frac{\pi a^2}{4} = \pi ab$

73) The smaller area between the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$ and the line $\frac{x}{3} + \frac{y}{4} = 1$ is

A) $\pi - 2$ B) $3(\pi - 2)$ C) $3(\pi + 2)$ D) $3(3\pi + 2)$

Correct Answer: B

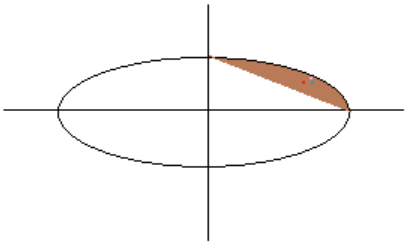
Solution: Area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab

Area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in first quadrant $\frac{\pi ab}{4}$

Area of the line bounded by the line $\frac{x}{a}+\frac{y}{b}=1$ with co-ordinate axes is $\frac{ab}{2}$

Smaller area between ellipse and line = $\frac{\pi ab}{4}-\frac{ab}{2}$

In the given problem $a=3, b=4$ area = $3\pi-6$



74) If $sin\theta, cos\theta, tan\theta$ are in G.P. then the value of $cot^6\theta - cot^2\theta =$ is

A) 1 B) 2 C) 4 D) 3

Correct Answer: A
Solution: $Sin\theta, Cos\theta, Tan\theta$ are in G.P then $Cos^2\theta = sin\theta tan\theta$

$Cos^2\theta = sin\theta \times \frac{sin\theta}{cos\theta} \Rightarrow cos^3\theta = sin^2\theta$

$cos^6\theta = sin^4\theta$

Dividing both sides with $sin^6\theta$ both sides

$cot^6\theta = \frac{1}{sin^2\theta} = cosecc^2\theta = 1 + cot^2\theta$

$cot^6\theta - cot^2\theta = 1$

75) If $f(x) = |x-1| + |x-2| + |x-3|$ then $f : [2,3] \to R$ is

A) One - one but not onto function B) An onto function only C) An identity function

D) Many to one function

Correct Answer: A
Solution: If $2 \le x \le 3$ then $f(x) = x - 1 + (x-2) - (x-3) = x$ is one -one becuase $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

Range $= [2,3] \neq R$

Hence $'f'$: $[2,3] \to R$ is not onto.
Hence 'f' is onne -one but not onto.

It is not identity function because domain and codomain are not same.

It is not many to one because different elements are not having the same image.

76) Two circles of equal radii 'r' cut orthogonally. If their centres are $(-2,-3)$ and $(-5,-6)$ then $r =$

A) 1 B) 2 C) 3 D) 4

Correct Answer: C
Solution: Distance between the centres $d = \sqrt{(-5+2)^2 + (-6+3)^2} = \sqrt{18}$

If two circles intersect orthogonally then $d^2 = r_1^2 + r_2^2$

$r_1 = r_2 = r$

$d^2 = 2r^2 = 18 \Rightarrow r = 3$

77) If 7 points out of 12 are in the same straight line, then the number of triangles formed is

A) 19 B) 158 C) 185 D) 201

Correct Answer: C
Solution: Every triangle is formed by selection of three points

Number of three letter selections that can be made from 12 points is ${}^{12}C_3$

Amon these 3 lettere selections fewof them are from collinear points which donot form a triangle.

Number of three letter selections that can be made from 7 collinear points are ${}^{7}C_3$

Hence number of triangles formedd from 12 points out of which 7 of them are collinear
$= {}^{12}C_3 - {}^{7}C_3 = 220 - 35 = 185$

78) The length of the common chord of the circles $x^2 + y^2 + 4x + 1 = 0$ and $x^2 + y^2 + 4y - 1 = 0$ is

A) $\sqrt{15/2}$ B) $\sqrt{15}$ C) $2\sqrt{15}$ D) $15\sqrt{2}$

Correct Answer: A
Solution: Radical axis of the circles $= S - S' = 0 \Rightarrow 4x - 4y + 2 = 0 \Rightarrow 2x - 2y + 1 = 0$

d = Distance from centre of the circle to the radical axis

Centre $(-2, 0)$ and radius $= \sqrt{4-1} = \sqrt{3}$

$d = |\frac{-4-0+1}{\sqrt{4+4}}| = \frac{3}{\sqrt{8}}$

Length of common chord of the circle = $2\sqrt{r^2 - d^2} = 2\sqrt{3 - \frac{9}{8}} = 2\frac{\sqrt{15}}{2\sqrt{2}} = \sqrt{\frac{15}{2}}$

79) If ${}^{n}c_{r-1} = 36, {}^{n}c_r = 84$ and ${}^{n}c_{r+1} = 126$ then

A) $n = 8, r = 4$ B) $n = 9, r = 3$ C) $n = 7, r = 5$ D) $n = 8, r = 2$

Correct Answer: B
Solution: ${}^{n}c_{r-1} = 36, {}^{n}c_r = 84$

$\Rightarrow \frac{{}^{n}c_r}{{}^{n}c_{r-1}} = \frac{84}{36} = \frac{7}{3}$

$\Rightarrow \frac{n-r+1}{r} = \frac{7}{3}$

$\Rightarrow 3n - 10r + 3 = 0 .....(1)$

${}^{n}c_{r+1} = 126, {}^{n}c_r = 84$

$\Rightarrow \frac{^nC_{r+1}}{^nC_r} = \frac{126}{84} = \frac{3}{2}$

$\Rightarrow \frac{n-r}{r+1} = \frac{3}{2}$

$\Rightarrow 2n - 5r - 3 = 0.....(2)$

Solving (1) and (2) we get $n = 9, r = 3$

80) The mean or average number of heads when we toss 10 unbiased coins is

A) 20 B) 10 C) 5 D) 15

Correct Answer: C

Solution: Probability of getting a head when one coin is tossed is $= p = \frac{1}{2}$

Mean $np = 10 \times \frac{1}{2} = 5$

81) Arrange the increasing order of dimensions of time of the following physcial quantities

A) decay constant

B) Intensity of radiation

C) Permitivity

D) Moment of inertia

A) A,B,D,C B) C,D,B,A C) B,A,D,C D) B,A,C,D

Correct Answer: C

Solution: A) $\lambda = \frac{0.693}{T_{\frac{1}{2}}} = T^{-1}$

B)$I = \frac{\rho}{A} = \frac{ML^2T^{-3}}{L^2} = MT^{-3}$

C)$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}; \varepsilon_0 = \frac{I^2T^2}{MLT^{-2}L^2} = M^{-1}L^{-3}I^2T^4$

D) $I = MR^2 = ML^2T^o$

The order of time in decay constant is $= -1$

The order of time in intensity of radiation is $= -3$

The order of time in permittivity is $= 4$

The order of time in the moment of inertia is $= 0$

$-3 < 1 < 0 < 4$

$\Rightarrow BADC$

82) The length of a rectangular plate is 10.52 m and its width is only 0.041 m. Find its area up to appropriate significant figures.

A) $0.4305\mathrm{m}^2$ B) $0.43\mathrm{m}^2$ C) $0.4\mathrm{m}^2$ D) $0.043\mathrm{m}^2$

Correct Answer: B

Solution: The length of a rectangular plate $\ell = 10.52m$ contains 4 significant figures.

The breadth of a rectangular plate $\ell = 0.041$ contains 2 significant figures.

In the number 0.041, the zero is called leading zero. The only function of leading zero is to fix the decimal point. Leading zeros are not counted as significant figures. Therefore, the number 0.041 has two significant figures.

When multiplying/dividing, the answer should have the same number of significant figures as the limiting term. The limiting term is the number with the least number of significant figures.

$A = l \times b = 10.52 \times 0.041 = 0.43132\, m^2$

Since b contains two significant digits product must contain two significant digits hence the final answer is 0.43$ since after second signifficant digit number is 1 which less than 5.

83) A particle is thrown with a speed u at an angle θ with the horizontal. When the particle makes an angle φ with the horizontal its speed changes to 'V' then

A) $V = u\cos\theta\cos\varphi$ B) $V = u\cos\theta\sec\varphi$ C) $V = u\cos\theta$ D) $V = u\cos\theta\sin\varphi$

Correct Answer: B
Solution: A particle is thrown with a velcoty u which makes an angle θ with horizontal then its horizontal velocity $u_x = u\ \cos\theta$

$u_y = u\ \sin\theta$

Let the velocity of the particle after some time be V.When the particle makes an angle φ with the horizontal then its horizontal velocity of the particle at that point be $V_x = V\cos\varphi$

The force of gravity does not affect the horizontal component of motion; a projectile maintains a constant horizontal velocity since there are no horizontal forces acting upon it.

$\Rightarrow u_x = V_x$

$u\cos\theta = V\cos\varphi$

$V = \frac{u\cos\theta}{\cos\varphi} = u\cos\theta\sec\varphi$

84) One train is approaching an observer at rest and another is receding him with same velocity 4 m/s. Both the trains blow whistles of same frequency of 243 Hz. The beat frequency in Hz as heard by the observer is: Speed of sound in air =320 m/s.

A) 10 B) 6 C) 4 D) 1

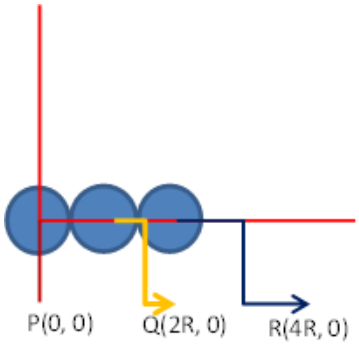
Correct Answer: B
Solution:

85) Three identical spheres each of radius 'R' are placed touching each other so that their centers A,B,C lie on a straight line.The position of center of mass from centre of mass of 'A' is

A) $\frac{2R}{3}$ B) $2\mathrm{R}$ C) $\frac{5R}{3}$ D) $\frac{4R}{3}$

Correct Answer: B

Solution:



Centre of mass $= (x_{cm}, y_{cm}) = (x_{cm}, 0)$ since all the centres of all spheres lie on x-axis.

$x_{cm} = \frac{m_1x_1+m_2x_2+m_3x_3}{m_1+m_2+m_3}$

$\frac{m\times0+m\times2R+m\times4R}{m+m+m}$

$= \frac{6mR}{3m} = 2R$

86) A tennis ball bounces down a flight of stairs striking each step in turn and rebounding to the half-height of the step. The coefficient of restitution is

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) $\frac{1}{\sqrt{2}}$ D) 1

Correct Answer: C

Solution: Let 'h' be the height of each step.

If a ball dropped falls freely $u = 0$

If it hits the ground with a velocity v

$\Rightarrow v^2 - u^2 = 2gh \Rightarrow v^2 = 2g(2h) = 4gh$

If it raises to height h ,at the point velocity = 0

$\Rightarrow 0^2 - v^2 = -2gh \Rightarrow v^2 = 2gh$

The ball falls under gravity by 2h and rebounds to a height 'h'

$e = \sqrt{\frac{h_1}{h}} = \sqrt{\frac{h}{2h}} = \frac{1}{\sqrt{2}}$

87) A boat of mass 80 kg is floating on still water. A dog of mass 20 kg on the boat is at a distance of 10m from the shore. The dog moves on the boat by a distance of 2m towards the shore. The distance of the dog from the shore is

A) 11.6 m B) 8.4 m C) 9.6 m D) 10.4 m

Correct Answer: B

Solution:

As the dog moves towards shore by 2m, the system i.e dog and boat move towards the left.

Mass of boat = M = 8Kg

Mass of dog = m = 20kg

Let shift in the centre of mass be x_1

Let $l$be the distance moved by the dog $\Rightarrow l = 2$

$\Rightarrow x_1 = \frac{ml}{M+m} = \frac{20\times2}{20+80} = 0.4m$

Distance from thedog to the boat $10 - 2 + 0.4 = 8.4m$

88) A suitcase is gengtly dropped on a conveyor belt moving at 3 m/s. If the coefficient of friction between the belt and the suitcase is 0.5. Find the displacement of suitcase relative to conveyor belt before the slipping between the two is stopped $(g = 10m/s^2)$

A) 2.7 m B) 1.8 m C) -0.9 m D) 1.2 m

Correct Answer: C

Solution: The initial velocity of box relative to The belt is 3m/s in backward direction .The box experiences maximum frictional force $F = \mu N = \mu mg$

acceleration=$= \frac{F}{m} = \frac{\mu mg}{m} = \mu g$

$a = 0.5 \times 10 = 5\, m/s^2$

Let the box comes to rest in distance 'S'

using $v^2 - u^2 = 2as$

$0 - 3^2 = +2 \times 5 \times S$

$-9 = 10S \ = S = -0.9\, m$

The negative sign represents the box moves in opposite direction of motion of belt.

89) The string of a pendulum, having bob of mass m is displaced through 90^o from the vertical and then released. What is the tension in the string when the bob is passing through the mean position?

A) mg B) 3mg C) 5mg D) 6mg

Correct Answer: B

Solution: The string of a pendulum, having bob of mass m is displaced through 90^o from the vertical then it is in a horizontal position.

Length of the simple pendulum $= \ell$

Potential energy when this string is horizontal $= mg\ell$

When bob comes to mean position its potential energy becomes zero. The total energy is converted to kinetic energy.

Kinetic energy when the string is vertical $= \frac{1}{2}mv^2$ where v is velocity of the bob in mean position.

$mgl = \frac{1}{2}mv^2$

$V = \sqrt{2gl}$

The circular motion of an equation at the mean position.

Cnetripetal forcce = Centrifugal force

Let T may be the tension in the string

$T - mg = \frac{mv^2}{r}$

$= \frac{m2g\ell}{\ell} + mg = 3mg$

90) A particle is executing uniform circular motion with angular momentum L. If its frequency of rotation is halved and KE is doubled its angular momentum becomes

A) L B) $2L$ C) $\frac{L}{2}$ D) $4L$

Correct Answer: D
Solution: Angular momentum of a particle $L = I\omega$ Where, I is the moment of inertia

Kinetic energy of a particle $E = \frac{1}{2}I\omega^2$

$\Rightarrow E = \frac{1}{2}\frac{L}{\omega} \times \omega^2 = \frac{1}{2}L\omega$

Angular frequency $\omega = 2\pi n$

$E = \frac{1}{2}L \times 2\pi n = \pi nL$

$\Rightarrow L \propto \frac{E}{n}$

$\Rightarrow L_2 = L_1 \times \frac{E_2}{E_1} \times \frac{n_1}{n_2}$

Given $n_2 = \frac{n_1}{2}, E_2 = 2E_1$

$\Rightarrow L_2 = L \times \frac{2E}{E} \times \frac{n}{\left(\frac{n}{2}\right)} = 4L$

91) A satellite is launched into a circular orbit of radius R around the earth. A second satellite is launched into an orbit of radius 1.01 R. The period of the second satellite is longer than that of the first by approximately.

A) 0.5% B) 1.0% C) 1.5% D) 3.0%

Correct Answer: C
Solution: By keplers third law $T^2 \alpha R^3$

$\frac{T_2}{T_1} = \left(\frac{1.01R}{R}\right)^{\frac{3}{2}}$

$= (1 + 0.01)^{\frac{3}{2}}$

$(1+x)^n \approx 1+nx$ if 'x' is very small

$\frac{T_2}{T_1} = 1 + 0.01 \times \frac{3}{2}$

$\frac{T_2}{T_1} - 1 = 0.015$

Percentage increase in the period

$= \frac{T_2 - T_1}{T_1} \times 100 = 0.015 \times 100 = 1.5\%$

92) A simple pendulum has time period T^1. The point of suppension is now moved upwards according to the relation $y = kt^2$, $(k = 1m/sec^2)$ where y is the vertical displacement. The time period now becomes 'T^2' then find the ratio of $\frac{T_1^2}{T_2^2}(g = 10m/sec^2)$

A) $\frac{5}{6}$ B) $\frac{6}{5}$ C) 1 D) $\frac{4}{5}$

Correct Answer: B

Solution: upward displacement ;$y = kt^2$

$= \frac{d^2y}{dt^2} = 2k$

or $a_y = 2\,m/s^2$ Given $k = 1\,m/s^2$

$T_1 = 2\pi\sqrt{\frac{l}{g}}$ and

$T_2 = 2\pi\sqrt{\frac{l}{g+a_y}}$

$\frac{{T_1}^2}{{T_2}^2} = \frac{g+a_y}{g} = \frac{10+2}{10} = \frac{12}{10} = \frac{6}{5}$

93) A metal rope of density $6000kgm^{-3}$ has a breaking stress $9.8 \times 10^8 \mathrm{Nm}^{-2}$ .This rope is used to measure the depth of the sea. Then the depth of the sea that can be measured without breaking is

A) $10 \times 10^3 m$ B) $20 \times 10^3 m$ C) $30 \times 10^3 m$ D) $40 \times 10^3 m$

Correct Answer: B

Solution: Rope densty $\rho_m = 6000kg/m^3$

Density of the water $\rho_w = 1000Kg/m^3$

Breaking stress $= l\rho_m g$

Net force on the upper end F_{net} = weight of the rope - force of buoyancy.

Stress$= \frac{F_{net}}{A} = l(d_{Rope} - d_{liquid})g$

Where ℓ is the depth of the see

$9.8 \times 10^8 = l(6000 - 1000)9.8$

$l = \frac{10^8}{5000} = 20 \times 10^3 m$

94) If W is the amount of work done in blowing a bubble of volume V, what will be the amount of work done to blow a bubble of volume 8 V?

A) $2\mathrm{W}$ B) $4\mathrm{W}$ C) $8\mathrm{W}$ D) $16\mathrm{W}$

Correct Answer: B
Solution: $W = T\Delta A$

$A = 4\pi r^2 \ = A\,\alpha\, r^2$

$V = \frac{4}{3}\pi r^3 \ = v\,\alpha\, r^3$

$W \alpha A$

$\alpha (V)^{\frac{2}{3}} \alpha\, (V)^{\frac{2}{3}}$

$\frac{W_1}{W_2} = \left(\frac{V_1}{V_2}\right)^{\frac{2}{3}}$

$= \left(\frac{V}{8V}\right)^{\frac{2}{3}} = \frac{1}{4}$

$W_2 = 4W$

95) Two raindrops reach the earth with different terminal velocities having a ratio of 16:9. The ratio of their volumes is

A) $4:3$ B) $64:27$ C) $16:9$ D) $3:4$

Correct Answer: B
Solution: Terminal velocity is the maximum velocity attainable by an object as it falls through a fluid. It occurs when the sum of the drag force F_d and the buoyancy is equal to the downward force of gravity F_g acting on the object. Since the net force on the object is zero, the object has zero acceleration.

g is the gravitational acceleration,

ρ is the density of the fluid,

σ is the density of the object,

η coefficient of viscosity.

r is the radius of spherical drop.

Terminal velocity $= V_t = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$

g is constant, fluid here is air, object is rain drop.only rain drops are of different size but σ, ρ are same

In both cases air is the fluid hence η is same

$\Rightarrow V_t \propto r^2$

V= volume of spherical air drop $\frac{4\pi r^3}{3}$

$\Rightarrow V \propto r^3 \Rightarrow r^2 \propto V^{\frac{2}{3}}$

$V_t \propto V^{\frac{2}{3}}$

The terminal velocity of first rain drop $= V_1$

The terminal velocity of the second rain drop $= V_2$

$\frac{V_1}{V_2} = \left(\frac{V_{t_1}}{V_{t_2}}\right)^{\frac{3}{2}}$

$\Rightarrow \left(\frac{16}{9}\right)^{\frac{3}{2}} = \frac{64}{27}$

96) During an experiment, an ideal gas is found to obey an additional law $VP^2 = constant$. The gas is initially at a temperature T and volume V. When it expands to 2V, the temperature becomes

A) $2T$ B) $\sqrt{2}T$ C) $\frac{T}{2}$ D) $\frac{T}{\sqrt{2}}$

Correct Answer: B

Solution: Given $VP^2 = const$

From ideal gas equation $P = \frac{RT}{V}$

$V\left(\frac{RT}{V}\right)^2 = const$

$\frac{T^2}{V} = const$

$\left(\frac{T_1}{T_2}\right)^2 = \frac{V_1}{V_2}$

$= \frac{V}{2V}$

$\frac{T_1}{T_2} = \frac{1}{\sqrt{2}}$

$T_2 = \sqrt{2}T$

97) An isolated tube traveling at a speed 'V' contains 'm' grams of gas of molecular weight 'M'. If the tube is suddenly stopped, the rise in temperature of a gas is

A) $\frac{MV^2(\gamma-1)}{2R}$ B) $\frac{m}{M}\frac{V^2(\gamma-1)}{2R}$ C) $\frac{mV^2\gamma}{2R}$ D) $\frac{MV^2\gamma}{2R}$

Correct Answer: A

Solution: n= number of moles of gas

Number of moles $n = \frac{Mass\ of\ the\ substance}{Molecular\ weight}$

$\Rightarrow n = \frac{m}{M}$

Energy available after the gas flow is suddenly stopped $\Delta E = \frac{mv^2}{2} = \frac{MnV^2}{2}$

Rise in internal enrfgy $\Delta U = nC_v dT$

Energy is utilised in raising the internal energy $\Rightarrow \Delta E = \Delta U$

Specific heat at constant volume $C_v = \frac{R}{\gamma-1}$

$\frac{MV^2}{2} = \frac{R}{(\gamma-1)}dT$

$\Rightarrow dT = \frac{MV^2(\gamma-1)}{2R}$

98) A Carnot engine whose sink is at 300K has an efficiency of 40.By how much should the temperature of source be increased so as to increase its efficiency by 50 of original efficiency.

A) $280K$ B) $275K$ C) $325K$ D) $250K$

Correct Answer: D
Solution: case(i)

$\eta = 1 - \frac{T_2}{T_1}$

$\frac{40}{100} = 1 - \frac{300}{T_1}$

$\frac{300}{T_1} = 0.6$

$T_1 = 500K$

case(ii):$\eta = 1 - \frac{T_2}{T_1}$

$(0.4 + 0.4 \times 0.5)0.6 = 1 - \frac{300}{T_1}$

$\frac{300}{T_1} = 0.4 \Rightarrow T_1 = 750k$

$\Delta T = 750 - 500 = 250K$

99) A body cools from 60^oC to 40^oC in 7 minutes. What can be its temperature after the next 7 minutes? The temperature of surroundings is 10^oC

A) 20^0C B) 38^0C C) 32^0C D) 28^0C

Correct Answer: D
Solution: By Newton's law of cooling, the temperature of a body changes at a rate proportional to the difference in temperature between the body and its surroundings.

$\frac{dT}{dt}\alpha\left(\frac{\theta_1+\theta_2}{2} - \theta_s\right)$

A body cools from 60^oC to 40^oC in 7 minutes.

$\Rightarrow \frac{60-40}{7}\alpha\left(\frac{60+40}{2} - 10\right)$

A body cools from 40^oC to θ^oC in next 7 minutes.

$\frac{40-\theta}{7}\alpha\left(\frac{40+\theta}{2} - 10\right)$

$\frac{20}{40-\theta} = \frac{40}{20+\theta} \times 2$

$20 + \theta = 160 - 4\theta$

$5\theta = 140$

$\theta = 28^\circ C$

100) A wire whose linear density is $5 \times 10^{-3} kg/m$ is strentched between two points with a tension 450 N.The wire resonates at a frequency of 420 Hz. The next heighest frequency at which the same wire resonates is 490 Hz. What is the length of the wire ?

A) 1.2 m B) 1.8 m C) 2.1 m D) 8.1 m

Correct Answer: C
Solution: $n_1 = \frac{P}{2l}\sqrt{\frac{T}{m}}$

$n_2 = \frac{(P+1)}{2l}\sqrt{\frac{T}{m}}$

$\frac{n_1}{n_2} = \frac{P}{P+1}$

$\frac{420}{490} = \frac{P}{P+1} \Rightarrow \frac{6}{7} = \frac{P}{P+1} \Rightarrow 6P + P = 7P$

p=6

$n_1 = \frac{6}{2l}\sqrt{\frac{450}{5\times10^{-3}}}$

$l = \frac{6}{2\times420} \times 300 = \frac{1800}{840} = 2.1\,m$

101) Two men stand a certain distance apart beside a long metal fence on a still day; one man places his ear against the fence while the other gives the fence a sharp knock with a hammer. Two sounds separated by a time interval of 0.5 second are heard by the first man. If the velocity of sound in air is $330\mathrm{ms}^{-1}$ and in the metal is $5280\mathrm{ms}^{-1}$, how far apart are the men ?

A) $352m$ B) $330m$ C) $165m$ D) $176m$

Correct Answer: D

Solution: $d = \frac{V}{t}$

In the case of metal d $= V_m\ t$

In the case of air d=$V_a = (t + 0.5)$

$V_m\, t = V_a(t + 0.5)$

$5280\, t = 330\,(t + 0.5)$

$t = \frac{0.5}{15}$ sec

$d = V_m \times t$

$= 5280 \times \frac{0.5}{15}$

$= 176\,m$

102) In a compound microscope, the object is 1 cm from the objective lens. The lenses are 30 cm apart and the intermediate image is 5 cm from the eyepiece. What magnification is produced?

A) 25 B) 50 C) 100 D) 125

Correct Answer: D

Solution: $m = \frac{v}{u}\left(1 + \frac{D}{f_e}\right)$

$= \frac{25}{1}\left(1 + \frac{D}{\left(\frac{25}{4}\right)}\right)$

$= 25 \times 5$

$= 125$

103) If a prism of refracting angle 60^0 and refractive index 2 is immersed in a liquid of refractive index $\sqrt{2}$ then the angle of minimum deviation would be

A) $\frac{\pi}{2}$ B) $\frac{\pi}{3}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{6}$

Correct Answer: D

Solution: $\mu_g{}^w = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

$\frac{2}{\sqrt{2}} = \sin\left(\frac{A+D}{2}\right) \times 2$

$\frac{A+D}{2} = \frac{\pi}{4}$

$\Delta = 90 - 60 = 30^0$

104) In an ideal double slit experiment, when a glass plate (refractive index 1.5) of thickness t is introduced in the path of one of the interfering beams (wavelength λ), the intensity at the position where the central maximum occurred previously remains unchanged. The minimum thickness of the glass plate is :

A) 2λ B) $\frac{2\lambda}{3}$ C) $\frac{\lambda}{3}$ D) λ

Correct Answer: A

Solution: Path difference due to slab sholud be integral multiple of λ

$\Delta x = n\lambda$

$(\mu - 1)t = n\lambda$ where n=1,2,3....

$t = \frac{n\lambda}{\mu - 1}$

for minimum of 't'

n=1

$t = \frac{\lambda}{\mu - 1} = 2\lambda$

105) A uniform electric field pointing in the positive x–direction exists in a region. Let A be the origin, B be the point on the x–axis at $x = +1cm$ and C be the point on the y–axis at $y = +1cm$. Then the potentials at A,B and C satisfy

A) $V_A < V_B$ B) $V_A > V_B$ C) $V_A < V_C$ D) $V_A > V_C$

Correct Answer: B

Solution: Electric lines of force are parallel in the case of emi form electric field.The points A and C are on the transverse pane whose electric potential is same at each and every point.

$V_A = V_C$

As electric lines of force are directed from higher potential to lower potential :

$V_B < V_A = V_C$

(or) $V_A > V_B$

106) P and Q are two points on the axial line and equatorial line of a short bar magnet at the same distance from its centre. If $\vec{B}_P$ and $\vec{B}_Q$ are the magnetic inductions at P and Q, then (magnetism and matter)

A) $\vec{B}_P = 2\vec{B}_Q$ B) $\vec{B}_P = -2\vec{B}_Q$ C) $\vec{B}_Q = 2\vec{B}_P$ D) $\vec{B}_Q = -2\vec{B}_P$

Correct Answer: B

Solution: $\overline{B}_a = -2\overline{B_e}$

The negative sign represents opposite direction.

107) A short magnet is arranged parallel to the meridian with its north pole pointing geographic south then neutral point is formed at a distance 'd'. The magnet is rotated by 90^0 in clockwise direction then the net induction at the same distance on its east side is .....(magnetism and matter)

A) 0 B) $\sqrt{2}B_H$ C) $2B_H$ D) $\sqrt{5}B_H$

Correct Answer: B

Solution: When N-pole of the bac magnet is pointly geopgraphic south null points are obtaines m its axial time,where

$B_a = B_H$

When the bar magnet is rotated by 90°

B_a and B_H are $\perp^u$ and equal

$B_r = \sqrt{{B_H}^2 + {B_H}^2} = \sqrt{2}B_H$

108) Three identical condensers are connected in different combinations using all three each time. Arrange the following cases in the decreasing order of effective capacity

a) all in series b) all in parallel

c) two in series and one parallel d) two in parallel and one in series

A) a,d,c,b B) a,b,c,d C) d,c,b,a D) b,c,d,a

Correct Answer: D

Solution: a) All in series $= \frac{C}{3}$

b) Three in parallel $C_e = 3C$

c) Two in series and one in parallel $= \frac{C}{2} + C = \frac{3C}{2}$

d) Two in parallel and one in series $= \frac{2C \times C}{2C + C} = \frac{2}{3}C$

b>c>d>a

109) An electric cable of copper has just one wire of a radius of 9mm. Its resistance is 10Ω.This single copper cable is to be replaced by another cable containing 10 well insulating copper wires each of radius 3mm the resistance of the new cable is

A) 7.5Ω B) 9Ω C) 9.5Ω D) 10Ω

Correct Answer: B

Solution: Resistance of the wire $R\alpha\frac{1}{r^2}$ where 'r' is radius of the wire.

$\Rightarrow \frac{R_1}{R_2}=\left(\frac{r_2}{r_1}\right)^2$

$\Rightarrow \frac{10}{R_2}=\left(\frac{3}{9}\right)^2$

$R_2=90\Omega$

As the cable has 10 such wires in parallel,The effective resistance center of the cable is

$R_{eff}=\frac{90}{10}=9\Omega$

110) You are given 48 cells each of emf 2 V and internal resistance 1Ω . How will you connect them so that the current through an external resistance of 3Ω is the maximum

A) 8 cells in series, 6 such groups in parallel B) 2 cells in series, 4 such group in parallel

C) 16 cells in series, 3 such groups in parallel D) 24 cells in series, 2 such groups in parallel

Correct Answer: B

Solution: mn=48

for current to be maximum mR=nr

$\frac{48}{n}\times 3=n\times 1$

$n=12$

$m=4$

i,e 12 cells in series and 4 groups in parallel.

111) The thermo e.m.f produced in a thermo couple is $3\ \mu V$ per degree. If the temperature of the cold junction is 20^0C and the thermo e.m.f is 0.3 mV, the temperature of the hot junction is

A) 80^0C B) 100^0C C) 120^0C D) 140^0C

Correct Answer: C

Solution: $T-20=\frac{emf}{emf/^0C}$

$T-20=\frac{0.3\times10^{-3}}{3\times10^{-6}}$

$T-20=100$

$T=120^0C$

112) An electron is accelerated to a high speed along the axis of a cathode ray tube by the application of a potential difference of V volts between the cathode and the anode. The particle then passes through a uniform transverse magnetic field in which it experience a force F. If the potential difference between the anode and the cathode is increased to 2V, the electron will now experience a force

A) $F/\sqrt{2}$ B) $\frac{F}{2}$ C) $\sqrt{2}F$ D) $2F$

Correct Answer: C
Solution: $W = Vq$

$\frac{1}{2}m\vartheta^2 = Vq$

$\vartheta \alpha \sqrt{V}$

$\frac{\vartheta_1}{\vartheta_2} = \sqrt{\frac{V}{2V}} \Rightarrow \vartheta_2 = \sqrt{2}\,\vartheta_1$

$F = Be\vartheta$

$F\,\alpha\,\vartheta$

$\frac{F_1}{F_2} = \frac{\vartheta_1}{\vartheta_2} = \frac{1}{\sqrt{2}}$

$F_2 = \sqrt{2}F$

113) A metallic square loop ABCD is moving in its own plane with velocity v in a uniform magnetic field perpendicular to its plane as shown in fig. An electric field is induced (EMI)

A) in AD, but not in BC B) in BC, but not in AD C) neither in AD nor in BC

D) in both AD and BC

Correct Answer: D

Solution: For both AD and BC: $\overline{B}, \bar{l}\ and\ \overline{\vartheta}$ are perpendicular ,therefore emf is induced in them.

But for both AB and DC:$\overline{R}\ and\ \overline{\vartheta}$ are parallel,there emf induced is zero.

114) An electron with speed v and a photon with speed c have the same de Broglie wavelength. If the kinetic energy and momentum of electrons is E_e and P_e and that of photon is E_{ph} and P_{ph} respectively, then the correct option is= (DUAL NATURE OF MATTER)

A) $\frac{E_e}{E_{ph}} = \frac{2c}{\vartheta}$ B) $\frac{E_e}{E_{ph}} = \frac{\vartheta}{2c}$ C) $\frac{P_e}{P_{ph}} = \frac{2c}{\vartheta}$ D) $\frac{P_e}{P_{ph}} = \frac{\vartheta}{2c}$

Correct Answer: B
Solution: $\lambda = \frac{h}{P} = \frac{h}{mv}$

$\frac{E_e}{E_{Ph}} = \frac{\frac{1}{2}m_e\vartheta^2}{m_{Ph}C^2}$

Given $m_eV = m_{Ph}C$

$\frac{E_e}{E_{Ph}} = \frac{1}{2}\frac{m_{Ph}C}{V} \times \frac{\vartheta^2}{m_{Ph}C^2} = \frac{\vartheta}{2C}$

115) A nucleus at rest splits into two nuclear parts having radii in the ratio 1 : 2. Their velocities are in the ratio

A) $8:1$ B) $6:1$ C) $4:1$ D) $2:1$

Correct Answer: A
Solution: Volume X density = Mass

Density is the same for both the fragments as both get generated from the same nucleus.

So, Mass of the fragments are in ratio of their volumes

Volume $\propto$ m

volume of sphere $v = \frac{4\pi r^3}{3}$

So, we have

$\frac{V_1}{V_2} = \frac{m_1}{m_2} = \frac{r_1^3}{r_2^3}$

$\Rightarrow \frac{m_1}{m_2} = \frac{1^3}{2^3}$

So, by conservation of momentum $m_1 v_1 = m_2 v_2$

So, the velocities are in ratio, $v_1 : v_2 = 8 : 1$

116) An optical fiber can offer a transmission bandwidth in excess of

A) 100 GHz B) 100 MHz C) 100 Hz D) 100 KHz

Correct Answer: A
Solution: An optical fibre can offer a transmission bandwidth in excess of 100 GHz

optical communication using fibres is performed in the frequency range of 1 THz to 1000 Thz micro waves to ultraviloet

117) A heating coil is labeled 100 W, 220 V. The coil is cut in two equal halves and the two pieces are joined in parallel to the same source. The energy now liberated per sec is

A) 200 J B) 400 J C) 25 J D) 50 J

Correct Answer: B
Solution: A heating coil is labeled 100 W, 220 V.

$\Rightarrow P = 100W, V = 220V$

Power $= P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P} = \frac{220 \times 220}{100} = 484\Omega$

Heat produced per second $P = \frac{H}{t} = \frac{V^2}{R} = \frac{220 \times 220}{484} = 100\, J/\sec$

The coil is cut in two equal halves then resistance of each wire is $\frac{R}{2}$

Two pieces are joined in parallel , then effective resistance $\frac{R}{2}$.

$R_2 = \frac{R}{4} = \frac{484}{4} = 121\Omega$

The energy now liberated per sec is

$\frac{H_2}{t} = \frac{V^2}{\left(\frac{R}{4}\right)} = 4 \times 100 = 400\, J/\sec$

118) A Parallel plate condenser of capacity 100 pF is connected to 230 V of AC supply of 300rad / sec frequency. The rms value of displacement current

A) $6.9\,\mu\,A$ B) $2.3\,\mu\,A$ C) $9.2\,\mu\,A$ D) $4.6\,\mu\,A$

Correct Answer: A
Solution: $I_{rms} = E_{rms}wc$
$= 230 \times 300 \times 100 \times 10^{-12}$
$= 6.9\,\mu A$

119) A parallel plate capacitor of plate separation 2 mm is connected in an electric circuit having source voltage 400V. If the plate area is $60\,cm^2$, then the value of displacement current for 10^{-6} sec. will be

A) 1.062 amp B) $1.062 \times 10^{-2}\,amp$ C) $1.062 \times 10^{-3}\,amp$ D) $1.062 \times 10^{-4}\,amp$

Correct Answer: B
Solution: $I_d = \frac{\varepsilon_o A}{d}\frac{dV}{dt}$
$= \frac{8.85\times10^{-12}\times60\times10^{-4}}{2\times`10^{-3}} \times \frac{400}{10^{-6}}$
$= 106.2 \times 10^{-4} A$
$= 1.062 \times 10^{-2} A$

120) A condenser has two conducting plates of radius 10cm seperated by a distance of 5mm. It is charged with a constant current of 0.15 A. The magnetic field at a point 2cm from the axis in the gap is

A) $6 \times 10^{-8}\,T$ B) $3 \times 10^{-8}\,T$ C) $6 \times 10^{-6}\,T$ D) $3 \times 10^{-6}\,T$

Correct Answer: A
Solution: $B = \frac{\mu_0}{2\pi} I_d \frac{r}{R^2}$
$= 2 \times 10^{-7} \times 0.15 \times \frac{2\times10^{-2}}{100\times10^{-4}}$
$= 0.6 \times 10^{-7}$
$= 6 \times 10^{-8} T$

121) EAN of M in $[M(CO)_x]$ is 36, the value of x is _____ atomic number of M is 26.

A) 4 B) 6 C) 5 D) 3

Correct Answer: C
Solution: Effective atomic number (EAN), a number that represents the total number of electrons surrounding the nucleus of a metal atom in a metal complex. It is composed of the metal atom's electrons and the bonding electrons from the surrounding electron-donating atoms and molecules.

EAN= Z metal – Oxidation state of the metal + 2×coordination number of the metal.

where Z =atomic number of the metal

The oxidation state of the metal= number of electrons lost during the formation of the metal ion from its atom

The coordination number of the metal= number of electrons donated by the ligands.

As Ni Fe Cr follows EAN, then,$M(CO)_x$ Oxidation state of the metal is 0

$\Rightarrow 36 = 26 - 0 + 2x \Rightarrow x = 5$

122) The ionization potential of X^- ion is equal in magnitude to

A) Electron affinity of X^+ B) Ionisation potential of X C) Ionisation potenital of X^+

D) Electron affinity of X

Correct Answer: D
Solution: $X^- \rightarrow X + e^-; \Delta H = I.P\, of X^-$

$X + e^- \rightarrow X^-; \Delta H = -E.A\, of X$

Comparing the above equations we get

I.P of X^- = E.A of X

123) Fractional bond order is in

A) O_2 B) O_2^+ C) O_2^{2-} D) N_2

Correct Answer: B
Solution:

no of e^-	Band order
16	2
15	2.5
14	3
13	2.5
12	2

124) In O_3 molecule, the formal charge of terminal oxygen and central oxygen are

A) -1,1 B) -1,0 C) 1,1 D) 1,0

Correct Answer: A
Solution: Number of valence electrons in $O_3 = 3 \times 6 = 18$

The normal skeleton structure of O_3 is O-O-O

To complete octet rule for central atom left and right Oxygen atoms are respectively 4, 6, 6

In normal skeleton structure, there are two covalent bonds,ie 4 electrons

Remaining electrons 18-4 = 14

The end O atom in O= O: It has six valence electrons in the free state. It has two lone pairs (or four nonbonding electrons) and two bonds(or four bonding electrons). Thus,

Free charge $= 6 - 4 - \frac{1}{2} \times 4 = 0$

But to complete octet we require 16 electrons. Hence we need one more covalent bond.

We can put a double bond between first two oxygen atoms or the last two oxygen atoms.

Considering one of the possibility O=O-O

Lewis structure of ozone molecule

$\ddot{\underset{..}{O}} = \ddot{O} - \ddot{\underset{..}{O}}$:

The formal charge on the central oxygen atom in O_3 molecule is +1. Formal charge in central O= valence electron − $\frac{1}{2}\times$ bonding electron −non-bonding electron $= 6 - \frac{1}{2} \times 6 - 2 = 1$

The end O atom in O= O : It has six valence electrons in the free state. It has two lone pairs or four nonbonding electrons and two bonds or four bonding electrons. Thus, Free charge
$= 6 - 4 - \frac{1}{2} \times 4 = 0$

The end O atom in O−O: It has six valence electrons, three lone pairs or six nonbonding electrons , and one bond or two bonding electrons. Thus,

Free charge $= 6 - 6 - \frac{1}{2} = -1$

125) $CaCO_3 \xrightarrow{\triangle} X + CO_2, X + 3C \rightarrow Y + CO, Y + 2H_2O \rightarrow Z + Ca(OH)_2$
Incorrect statement about Z is

A) It contains two acidic hydrogens B) with S_2Cl_2 it forms mustard gas

C) it is used in Hawker's lamp

D) it has garlic smell due to the presence of impurities hydrogen sulphide and phosphine

Correct Answer: B
Solution: $CaCO_3 \rightarrow CaO + CO_2$

$CaO + 3C \rightarrow CaC_2 + CO$

$CaC_2 + 2H_2O \rightarrow C_2H_2 + Ca(OH)_2$

Hence $Z = C_2H_2$ which contains two acidic hydrogens.Hence option A is correct.

Acetylene is not especially toxic, but when generated from calcium carbide, it can contain toxic impurities such as traces of phosphine and arsine, which give it a distinct garlic-like smell.

Hence option D is correct.

Lamps that are used by hawkers use Acetylene gas as a source of light which produces a very bright light.

Hence option C is correct.

Mustard gas is prepared from ethylene not from acetylene.

$CH_2 = CH_2 + S_2Cl_2 \rightarrow Cl - CH_2 - CH_2 - S - CH_2CH_2Cl + S$

mustard gas

126) In the following sequence of reactions the product (D) is
$CH \equiv CH \xrightarrow{HBr} A \xrightarrow{HBr} B \xrightarrow{alcKOH} C \xrightarrow{NaNH_2} D.\ D\ is$

A) Ethanol B) Ethyne C) Ethanal D) Ethene

Correct Answer: B

Solution: $CH \equiv CH \overset{HBr}{\longrightarrow} CH_2 = CHBr \overset{HBr}{\longrightarrow} CH_3 - CHBr_2$

$\overset{alcKOH}{\longrightarrow} CH_2 = CHBr \overset{NaNH_2}{\longrightarrow} CH \equiv CH$

127) The single bond between sulphur atoms is present in

A) $H_2S_2O_3$ B) $H_2S_2O_7$ C) $H_2S_2O_2$ D) $H_2S_2O_6$

Correct Answer: D

Solution: Structure of $H_2S_2O_7$ which does not contain bond between sulphur atoms.

$$OH - \overset{\overset{O}{\|}}{\underset{\underset{O}{\|}}{S}} - O - \overset{\overset{O}{\|}}{\underset{\underset{O}{\|}}{S}} - OH$$

Option B is wrong.

Structure of $H_2S_2O_3$ which does not contain bond between sulphur atoms.

$$OH - \overset{\overset{S}{\|}}{\underset{\underset{O}{\|}}{S}} - OH$$

Which doesn't contain single bond between sulphur atoms.

Structure of $H_2S_2O_4$ Does not contain single between sulphur atoms.

$$OH - \overset{\overset{O}{\|}}{S} = \overset{\overset{O}{\|}}{S} - OH$$

Structure of $H_2S_2O_8$ contain single bond between sulphur atoms.

$$OH - \overset{\overset{O}{\|}}{\underset{\underset{O}{\|}}{S}} - \overset{\overset{O}{\|}}{\underset{\underset{O}{\|}}{S}} - OH.$$

128) The expression for $'P_c'$ of gas can be

A) $\frac{PV}{RT}$ B) $3b$ C) $\frac{a}{27\,b^2}$ D) $\frac{8a}{27\,Rb}$

Correct Answer: C

Solution: The vander waals equation is written for 1 mole of a gas as

$(p + \frac{a}{V^2})(V - b) = RT$

After simplification we get $V^3 - \frac{(b+RT)}{P}V^2 + \frac{a}{b}V + \frac{ab}{p} = 0$

For critical values $V^3 - \frac{(b+RT)}{P}V^2 + \frac{a}{b}V + \frac{ab}{p} = (V - V_c)^3$

$\Rightarrow 3V_c = b + \frac{RT_c}{P_c}, 3V_c^2 = \frac{a}{P_c} and\, V_c^3 = \frac{ab}{P_c}$

$\Rightarrow P_c = \frac{a}{27b^2} : T_c = \frac{8a}{27Rb} : V_c = 3b$

129) Which of the following is an aromatic compound

A) Phenol B) Naphthalene C) Pyridine D) All

Correct Answer: D
Solution:

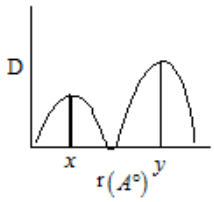
130) A hydrogen molecule and helium atom are moving with the same velocity. Then the ratio of their de-Brogile wave length is

A) 1:1 B) 4:1 C) 1:4 D) 2:1

Correct Answer: D
Solution: $\frac{\lambda_1}{\lambda_2} = \frac{M_2}{M_1}$

$\frac{\lambda_1}{\lambda_2} = \frac{4}{2} = \frac{2}{1} = 2:1$

131) The distance between x and y in the radial distribution curve drawn for 2s orbital is



A) $1.1\ A^0$ B) $2.6\ A^0$ C) $0.53\ A^0$ D) $2.07\ A^0$

Correct Answer: D
Solution: The radial distribution curve gives an idea about the electron density at a radial distance from the nucleus.

For a 2s orbital the radial node is at a distance of $2.6A^o$.

$x = 0.53A^o, y = 2.6A^o\ y - x = 2.6A^o - 0.53A^o = 2.07A^o$

132) Which of the reaction take place during isolation of inert gases from air by Fischer - Ringes method

a) $CaC_2 + N_2 \xrightarrow{1073K} CaCN_2 + C$

b) $CuO + CO \rightarrow Cu + CO_2$

c) $CO_2 + 2KOH \rightarrow K_2CO_3 + H_2O$

d) $N_2 + 3Mg \rightarrow Mg_3N_2$

A) all the above B) only d C) only a D) a,b,c only

Correct Answer: D
Solution: In Fischer's Ringe's method, air free from moisture and CO_2 is passed over a heated mixture 800^oC of 90% CaC_2 + 10 % $CaCl_2$ in an iron tube, when following reactions take place.

$CaC_2 + N_2 \overset{800^oC}{\longrightarrow} CaCN_2 + C$

$2C + O \rightarrow 2CO$

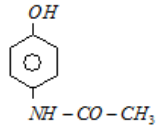
$C + O_2 \rightarrow CO_2$

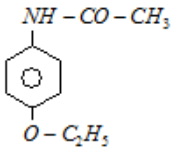
$2CaC_2 + 3CO_2O \rightarrow 2CaCO_3 + 5C$

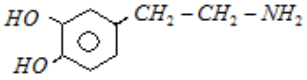
$CuO + CO \rightarrow Cu + CO_2$

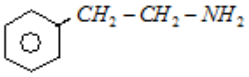
CO_2 gas is now absorbed by KOH solution.Thus,a mixture of inert gas is obtained.

133) The structure of phenacetin is

A) 

B) 

C) 

D) 

Correct Answer: B
Solution:

134) Gutta percha is

A) Cis - 1,4 - Poly isoprene B) Trans - 1,2 - poly isoprene C) Cis - 1,2 - poly isoprene

D) Trans - 1,4 - poly isoprene

Correct Answer: D
Solution:

135) A vessel contains a mixture of equal weights of CH_4, O_2 and SO_2 at a pressure of 600mm of Hg. The partial pressure of CH_4 in mm is

A) 34.28 B) 342.8 C) 171.4 D) 85.7

Correct Answer: B
Solution: Let us take 'x' gm of each of the components.

Molecular mass of $SO_2 = 64$

$\therefore$, Moles of $SO_2 = \frac{x}{64}$

Molecular mass of $CH_4 = 16$

$\therefore$ Moles of $CH_4 = \frac{x}{16}$

Molecular mass of $O_2 = 32$

$\therefore$ Moles of $O_2 = \frac{x}{32}$

Total moles = $\frac{x}{64} + \frac{x}{16} + \frac{x}{32} = \frac{7x}{64}$

Mole fraction of $CH_4 = \frac{\frac{x}{16}}{\frac{7x}{64}} = \frac{4}{7}$

Hence, partial pressure of CH_4 = mole fraction $\times$total pressure = $\frac{4}{7} \times 600 = 342.8\, mm\, of Hg$

So, the correct option is B

136) Regarding V_A group hydrides, incorrect one is

A) Ease of formation : $NH_3 > PH_3 > AsH_3 > SbH_3 > BiH_3$

B) Stability order : $BiH_3 > SbH_3 > AsH_3 > PH_3 > NH_3$

C) Boiling point order : $PH_3 < AsH_3 < NH_3 < SbH_3$

D) Diphosphine is the impurity present in trace amounts in phosphine

Correct Answer: B
Solution:

137) If $\triangle H^0$ and $\triangle S^0$ of a reaction are –180 kJ/mole and $-60 Jk^{-1}\, mole^{-1}$.The reaction becomes spontaneous

A) at 3000 K B) below 3000 K C) above 3000 K D) spontaneous at all temperature

Correct Answer: B
Solution: Gibb's Free energy of reaction ΔG

The change in the enthalpy (H) of the system minus the product of the temperature (Kelvin) and the change in the entropy (S) of the system:

$\Delta G = \Delta H - T\Delta S$

change in the free energy of the reaction: spontaneous if $\Delta G < 0$ for spontaneous

$\Delta G = \Delta H - T\Delta S = -180000 - T(-60) = -180000 + 60T < 0$

$\Rightarrow \Delta G < 0$

$\Rightarrow T < 3000K$

138) For the reaction $2HI(g) \rightleftharpoons H_2(g) + I_2(g)$ the degree of dissociation (α) of HI(g) is related to the equilibrium constant K_p , by expression.

A) $\frac{1+2\sqrt{K_p}}{2\sqrt{K_p}}$ B) $\sqrt{\frac{1+2K_p}{2}}$ C) $\sqrt{\frac{2K_p}{1+2K_p}}$ D) $\frac{2\sqrt{K_p}}{1+2\sqrt{K_p}}$

Correct Answer: D
Solution: $HI(g) \rightleftharpoons \frac{1}{2}H_2(g) + \frac{1}{2}I_2(g)$

$CaO \,\&\, CO_2$ $\frac{\alpha}{2}$ $\frac{\alpha}{2}$

$K_p = \frac{\left[\left(\frac{\alpha}{2}\right)P_T\right]^2}{(1-\alpha)^2 {P_T}^2}$

$\frac{\alpha}{1-\alpha} = 2\sqrt{K_p} \therefore \alpha = \frac{2\sqrt{K_p}}{1+2\sqrt{K_p}}$

139) E^0 of Fe^{+2}/Fe is –0.44V and E^0 of Cu^{+2}/Cu is +0.34V. Then in the cell

A) Cu oxidises Fe^{+2} ion B) Cu^{+2} oxidises iron C) Cu reduces Fe^{+2} ion

D) Cu^{+2} ion reduces Fe

Correct Answer: B
Solution: Element with Low Reduction potential undergoes oxidation.

140) Zn is used to protect corrosion of iron because

A) E^o_{ox} of Zn < E^o_{ox} of Fe B) E^o_{red} of Zn <E^o_{red} of Fe C) Zn is cheaper than Fe

D) Zn is abundantly available

Correct Answer: B
Solution: The rusting of iron can be prevented by coating the metal surface with another metal such as Zinc, which is oxidized in preference to iron.

Zn is a stronger reducing agent than Fe and thus can be oxidized easily than Fe.

$Fe^{2+}_{(}aq) + 2e^- \rightarrow Fe(s) \quad E^o = -0.44V$

$Zn^{2+}_{(}aq) + 2e^- \rightarrow Zn(s) \quad E^o = -0.763V$

141) Incorrect statement among the following is

A) Lyophilic sols are stable B) Low molecular weight inorganic sols are lyophobic

C) Starch solution is lyophilic sol D) Cloud is lyophilic sol

Correct Answer: D
Solution: The stability of lyophilic sols is a result of two factors, the presence of a charge and the solvation of colloidal particles. On the other hand, the stability of lyophilic sols is only because of the presence of a charge.

Hence option B is correct.

Lyophilic colloids are liquid loving colloids (Lyo means solvent and philic means loving.

Starch forms lyophilic sol when water is used as the dispersion medium. The formation of sol is accelerated by heating. Starch sol can be prepared by heating it and water at 100 °C.

Hence option C is correct.

Lyophobic sols -These sols are not created simply by combining the substances with the medium for dispersion. These are not stable, i.e. when a small amount of electrolyte is applied to them or when they are heated or shaken, these sols are easily coagulated. Lyophobic sols can be stabilized due to the adsorption of ions by the dispersed particles. They are irreversible in nature.

Examples of lyophobic sols include sols of metals and their insoluble compounds like sulfides and oxides.

Cloud is an aerosol. It is a colloidal solution suspension of water droplets in the air. It is a lyophobic colloid. Droplets in cloud precipitate as rain. It is a liquid in gas solution. The dispersion phase is droplets of water(liquid) and the dispersion medium is air (gas).

Hence option d is wrong.

142) In the equilibrium, $NH_4HS_{(S)} \rightleftharpoons NH_{3(g)} + H_2S(g)$.. The forward reaction can be favoured by

A) Adding some more NH_4HS B) Adding some more H_2S C) Adding some more NH_3

D) Removing some ammonia from the reaction mixture

Correct Answer: D
Solution: according to Le-chatelier principle.add of Reactanants or Removal of products Reaction moves in forward direction.

143) A solid has a structure in which 'W' atoms are located at the corners of a cubic lattice, 'O' atoms at the centre of edges and 'Na' atoms at the centre of the cube. The formula for the compound is

A) $NaWO_2$ B) $NaWO_3$ C) Na_2WO_3 D) $NaWO_4$

Correct Answer: B
Solution: In a unit cell, W atoms are at eight corners of the cube. Each corner atom contributes one eight to the unit cell. The contribution of W atoms to the unit cell = $\frac{1}{8} \times 8 = 1$O-atoms at the center of 12 edges.

Each edge center atom contributes one fourth to the unit cell.The contribution of O atoms to the unit cell = $\frac{1}{4} \times 12 = 3$

Na atom is at the centre of the cube. Body centre atom contributes one to the unit cell. The contribution of Na atoms to the unit cell =1

The ratio of number of atoms in the cube $W : O : Na = 1 : 3 : 1$

Hence, formula $= NaWO_3$

144) Silicon tetrafluoride on hydrolysis gives

A) Ortho silicic acid and meta silicic acid B) Meta silicic acid and silica

C) Meta silicic acid and hydro hexafluorosilicate D) Ortho silicic acid and hydro fluoro silicic acid

Correct Answer: C
Solution: The most characteristic reaction of silicon tetrafluoride is that with water, by which gelatinous silica and hydrofluosilicic acid (H2SiF6) are formed. The first reaction is one of hydrolysis:

$SiF_4 + 3H_2O = H_2SiO_3 + 4HF$

but this hydrolysis is not perfect, for silicon tetrafluoride combines with hydrogen fluoride to form hydrofluosilicic acid, H_2SiF_6 which can exist in aqueous solution:

$2SiF_4 + 4HF = 2H_2SiF_6$

consequently the complete reaction is

$3SiF_4 + 3H_2O = H_2SiO_3 + 2H_2SiF_6$

Silicon tetrafluoride react with water to produce metasilicic acid and hydro hexafluorosilicate

145) According to octet rule SO_3 contains -----number of dative bonds

A) 1 B) 2 C) 3 D) 4

Correct Answer: B
Solution:

146) $[Co(tn)_2Cl_2]Cl$ and $[Co(pn)_2Cl_2]Cl$ are

A) Hydrate isomers B) Ligand isomers C) Ionisation isomers D) Geometrical isomers

Correct Answer: B
Solution: Ligand isomerism is a type of structural isomerism in coordination complexes that arises from the presence of ligands which can adopt different isomeric forms.

147) The solution containing 6.8 grams of non-ionic solute in 100 grams of water was found to freeze at $-0.93^\circ C$.. If K_f for water is 1.86, the molar mass of solute is

A) 13.6 B) 68 C) 34 D) 136

Correct Answer: D
Solution: Freezing point depression = $\Delta T = 0.92^o$

According to Raoullts law $\Delta T_f = iK_fm$

i = van't Hoff factor = 1 for a non ionic solute

$\Rightarrow T_f^o - T_f = K_fm$

$\Rightarrow 0^oC - (-0.93^oC) = 1.86 \times m$

$\Rightarrow m = \frac{1}{2}$

$\Rightarrow \frac{Number\ of\ moles\ of\ solute}{Weight of solvent in Kg} = \frac{1}{2}$

$\Rightarrow \frac{\frac{6.8}{Molecular weight}}{\frac{100}{1}000} = \frac{1}{2}$

$\Rightarrow \frac{6.8\times1000}{M.W\times100} = \frac{1}{2}$

$\Rightarrow M.W = 136$

$\Rightarrow$ Molar mass of the solute =136gm/mole

148) The rate constant, Activation energy and the Arrhenius parameter of a chemical reaction at $25^\circ C$ are $3 \times 10^{-4} s^{-1}$, 104.4 kj/mole and $6 \times 10^{14} s^{-1}$ respectively.The value of the rate constant as $T \to \infty$ is

A) $2 \times 10^{18} s^{-1}$ B) $6 \times 10^{14} s^{-1}$ C) Infinity D) $3.6 \times 10^{30} s^{-1}$

Correct Answer: B

Solution: Arrhenius equation $K = Ae^{\frac{-Ea}{RT}}$

As $T \to \infty \Rightarrow RT \to \infty$

$\Rightarrow \frac{-Ea}{RT} \to 0$

$\Rightarrow e^{\frac{-Ea}{RT}} \to 1$

Hence $K \to A \, as \, T \to \infty$

$\therefore$ Value of K as $T \to \infty = 6.0 \times 10^{14} S^{-1}$

149) The weight of $NaOH$ needed to prepare 500ml. solution of $p^H = 12$ is

A) 0.4 g B) 0.2 g C) 12 g D) 6 g

Correct Answer: B

Solution: $[OH^-] = 10^{-2}$

$m = \frac{w}{mw} \times \frac{1000}{v \, in \, ml}$

$w = \frac{0.01 \times 40 \times 500}{1000} = 0.2g$

150) The K_a value of a week acid is 10^{-6}. The p^H of the buffer solution obtained by adding 0.2 moles of its salt with a strong base to one - litre of 0.1M solution of the acid is

A) 6.3 B) 7.7 C) 7 D) 4.5

Correct Answer: A

Solution: The expression for the pH of the acidic buffer solution is as given below.

$P^H = PKa + \log \frac{[Salt]}{[Acid]}$

$pKa = -logKa = -log10^{-6} = 6$

$P^H = 6 + \log \frac{[0.2]}{[0.1]} = 6 + log2 = 6.3$

151) $8Al + 30HNO_3 \to 8Al(NO_3)_3 + 3NH_4NO_3 + 9H_2O$ As per the above balanced

equation the number of moles of Al that can reduce 1 mole HNO_3 is

A) $\frac{3}{8}$ B) $\frac{16}{3}$ C) $\frac{9}{8}$ D) $\frac{8}{3}$

Correct Answer: D

Solution:

152) $C_6H_5OH + NH_3 \overset{Y/300^\circ C}{\longrightarrow} C_6H_5NH_2 \overset{X/200^\circ C}{\longrightarrow} C_6H_5Cl + 2NH_3$ X and Y are

A) $ZnCl_2, Cu_2O$ B) $Cu_2O, ZnCl_2$ C) $CuCl_2, ZnO$ D) $Cu_2Cl_2, ZnCl_2$

Correct Answer: B
Solution: $X = Cu_2O\ Y = ZnCl_2$

153) 2 – Butanol can be obtained by the hydrolysis of the addition product formed between

A) $HCHO, C_3H_7MgBr$ B) CH_3COH_3, C_2H_5MgI C) CH_3CHO, C_2H_5MgBr
D) CH_3CHO, CH_3MgCl

Correct Answer: C
Solution: 2-Butanol, or sec-butanol, is an organic compound with formula $CH_3CH(OH)CH_2CH_3$

Grignard reagent with aldehydes reacts together to give organo-metallic compounds which on hydrolysis result formation of alcohol.

$$CH_3CHO + CH_3CH_2MgBr \rightarrow CH_3CH_2\overset{CH_3}{\overset{|}{C}}HOH$$

154) If $CH_3COOH + NH_3 \longrightarrow X \underset{\Delta}{\longrightarrow} Y + H_2O$ then X and Y are respectively

A) $CH_3CONH_2; CH_4$ B) $CH_3COONH_4; CH_3CONH_2$
C) $CH_3CONH_2; CH_3COOH$ D) $CH_3NH_2; CH_3CONH_2$

Correct Answer: B
Solution: Ammonium acetate is a salt of weak acid acetic acid and weak base ammonia.

$CH_3COOH + NH_3 \rightarrow CH_3COONH_4 \overset{\Delta}{\longrightarrow} CH_3CONH_2$ (Ethanmide)

155) P^H value for a neutral amino acid (x) at an isoelectric point is 5.8. Now its solubility at this point in water is

A) Maximum B) Minimum C) Zero D) Unpredictable

Correct Answer: B
Solution: isoelectric point: The isoelectric point is the point at which the overall charge of the protein is zero.

If the pH of the solution is such that a particular molecule carries no net electric charge, the solute often has minimal solubility and precipitates out of the solution.

At isoelectric point solubility is minimum

156) In the Hydrogen atom as ‘n’ value increases the distance and energy difference between adjacent orbits respectively

A) Decreases, Decreases B) Increases, Decreases C) Increases, Increases

D) Decreases, Increases

Correct Answer: B

Solution: Radius of an orbit $= 0.529 \times \frac{n^2}{Z}$

where Z = 1for hydrogen

Therefore, difference in distance between orbit 2 and orbit 1

$D_1 = 0.529(2^2 - 1^2) = 0.529 \times 3A^o$

Also, the difference in distance between orbit 3 and orbit 2 ;

$D_2 = 0.529 \times (3^2 - 2^2) = 0.529 \times 5A^o$

Hence the distance between adjacent orbitals increases away from the nucleus in Bohr's model of the hydrogen atom.

$E_n = \frac{-13.6}{n^2}$

Energy value of $E_1 = 13.6ev$

Energy value of $E_2 = 3.4ev$

Energy value of $E_3 = 1.51ev$

Energy value of $E_4 = 0.85ev$

So $E_2 - E_1 = 10.2ev$

$E_3 - E_2 = 1.89$

$E_4 - E_3 = 0.66$

and 10.2 > 1.89 >0.66

So that $E_2 - E_1 > E_3 - E_2 > E_4 - E_3$ is correct order.

157) In the Fischer’s esterification process, the cleavage of bonds

A) $O - H of\, C_2H_5OH and C - O\, of\, CH_3COOH$

B) $C - O\ of\, C_2H_5OH\ and\, O - H\, of\, CH_3COOH$

C) $O - H\, of\, C_2H_5OH\, and\, C - C\, of\, CH_3COOH$

D) $C - C\, of\, C_2H_5OH\, and\, O - C\, of\, CH_3COOH$

Correct Answer: A

Solution:

158) The iron obtained from blast furnace is

A) Pig iron B) Soft iron C) Steel D) Wrought iron

Correct Answer: A

Solution: The iron obtained from blast furnace contains 4% Carbon and many impurities like S, P, Si, and

Mn in smaller amount this iron is known as pig iron.

159) The number of ATP molecules produced per 1 molecule of glucose in the biochemical reaction is

A) 18 B) 36 C) 54 D) 9

Correct Answer: B
Solution: Cellular respiration uses one molecule of glucose to produce 36 ATP molecules. In cellular respiration, energy is stored chemical bonds of glucose. The energy of glucose is known as ATP. Cells then must use this ATP to function. Therefore, the act of cellular respiration takes both stored glucose, and these ATP molecules to function correctly.

The glucose is oxidized during this process. Oxygen becomes reduced and forms water. Carbon dioxide is released as sugar. Two ATP molecules are produced. If oxygen is present in the pathway, 34 more ATP molecules are formed during this entire cellular respiration process.

Therefore 36 ATP molecule is produced during oxidation of 1 glucose molecule.

160) The compound that denotates on rubbing is

A) P_4O_{10} B) NO_2 C) XeO_3 D) XeF_4

Correct Answer: C
Solution: $6XeF_4 + 12H_2O \longrightarrow 4Xe + 2XeO_3 + 24HF + 3O_2$