

## RB IIT Academy EAMCET ENG MOCK TEST 1

1) The circles  $x^2 + y^2 = 25$  and  $x^2 + y^2 - 18x + 24y + 125 = 0$

A) Touch internally    B) Touch externally    C) Cut orthogonally    D) Intersect each other

Correct Answer: B

Solution: Equation of circles are  $x^2 + y^2 = 25$  and  $x^2 + y^2 - 18x + 24y + 125 = 0$

$C_1, C_2$  are centres of two circles  $C_1 = (0, 0), C_2 = (9, -12)$

$r_1, r_2$  are radii of two circles

then  $r_1 = 5$ , and  $r_2 = \sqrt{81 + 144 - 125} = 10$

$C_1C_2 = \sqrt{81 + 144} = 15$

$C_1C_2 = r_1 + r_2$

Hence circles touch externally

2) Range of  $\sqrt{x-2} + \sqrt{3-x}$

A)  $[1, \sqrt{3}]$     B)  $[1, \sqrt{2}]$     C)  $[1, 2]$     D)  $[2, 3]$

Correct Answer: B

Solution: Let  $y = \sqrt{x-2} + \sqrt{3-x} \Rightarrow y^2 = x-2 + 3-x + 2\sqrt{(x-2)(3-x)}$ .

$\Rightarrow y^2 = 1 + 2\sqrt{(x-2)(3-x)} \Rightarrow y^2 = 1 + 2\sqrt{-x^2 + 5x - 6}$

Maximum of  $-x^2 + 5x - 6 = \frac{24-25}{-4} = \frac{1}{4}$

$\sqrt{-x^2 + 5x - 6} \geq 0$ .

Hence  $0 \leq \sqrt{-x^2 + 5x - 6} \leq \frac{1}{2}$ .

$\Rightarrow 1 \leq 1 + 2\sqrt{-x^2 + 5x - 6} \leq 2$

$\Rightarrow 1 \leq y^2 \leq 2$

$y^2 - 1 \geq 0$  and  $y^2 - 2 \leq 0$

$y \in (-\infty, -1] \cup [1, \infty)$  and  $y \in [-\sqrt{2}, \sqrt{2}]$

$y \in [-\sqrt{2}, -1] \cup [1, \sqrt{2}]$  but  $y$  is always non-negative.

Hence  $y \in [1, \sqrt{2}]$

3)  $\vec{a}$  and  $\vec{b}$  are unit vectors along OA and OB. OC bisects the angle AOB. The unit vector along OC is

A)  $\frac{\vec{a}+\vec{b}}{|\vec{a}+\vec{b}|}$     B)  $\frac{\vec{a}-\vec{b}}{|\vec{a}-\vec{b}|}$     C)  $\frac{\vec{a}+\vec{b}}{2|\vec{a}+\vec{b}|}$     D)  $\frac{\vec{a}-\vec{b}}{2|\vec{a}-\vec{b}|}$

Correct Answer: A

Solution:  $\vec{OA} = \vec{a}, \vec{OB} = \vec{b}$ .

Vector along angle bisector of AOB is  $t\left(\frac{\vec{a}}{|\vec{a}|} + \frac{\vec{b}}{|\vec{b}|}\right)$

If  $\vec{a}, \vec{b}$  are unit vectors then  $|\vec{a}| = 1, |\vec{b}| = 1$

Unit vector along OC =  $\frac{t(\vec{a}+\vec{b})}{t|\vec{a}+\vec{b}|} = \frac{(\vec{a}+\vec{b})}{|\vec{a}+\vec{b}|}$

4)  $\vec{v} = 2\vec{i} + \vec{j} - \vec{k}$  and  $\vec{w} = \vec{i} + 3\vec{k}$ . If  $\vec{u}$  is a unit vector then the maximum value of the scalar triple product  $[\vec{u} \vec{v} \vec{w}]$  is

- A) -1    B)  $\sqrt{10} + \sqrt{6}$     C)  $\sqrt{59}$     D)  $\sqrt{60}$

Correct Answer: C

$$\text{Solution: } \vec{v} \times \vec{w} = \begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ 1 & 0 & 3 \end{vmatrix} = 3\vec{i} - 7\vec{j} - \vec{k}$$

$$|\vec{v} \times \vec{w}| = \sqrt{3^2 + 7^2 + 1} = \sqrt{59}$$

$$[\vec{u} \vec{v} \vec{w}] = (\vec{v} \times \vec{w}) \cdot \vec{u} = |\vec{v} \times \vec{w}| |\vec{u}| \cos \theta = \sqrt{59} \times 1 (\cos \theta) \leq \sqrt{59}$$

$$5) (\vec{a} \times \vec{b}) \cdot \vec{c} = |\vec{a}| |\vec{b}| |\vec{c}| \Rightarrow$$

- A)  $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a}$     B)  $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$     C)  $\vec{a} \perp \vec{b}, \vec{b} = \vec{c}$     D)  $\vec{a} \cdot \vec{b} = 1$

Correct Answer: B

Solution: Let  $\theta$  be angle between  $\vec{a} \times \vec{b}$  and  $\vec{c}$

$$[\vec{a} \vec{b} \vec{c}] = |\vec{a} \times \vec{b}| |\vec{c}| \cos \theta$$

Let  $\alpha$  be angle between  $\vec{a}$  and  $\vec{b}$

$$|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin \alpha \Rightarrow \sin \alpha \times \cos \theta = 1$$

$$\Rightarrow \sin \alpha = 1 \text{ and } \cos \theta = 1$$

$$\Rightarrow \alpha = \frac{\pi}{2} \Rightarrow \vec{a} \cdot \vec{b} = 0 \text{ and } \cos \theta = 1 \Rightarrow \theta = 0$$

$\theta = 0 \Rightarrow \vec{a} \times \vec{b}$  is parallel to  $\vec{c}$

$\Rightarrow \vec{a}$  and  $\vec{b}$  is perpendicular to  $\vec{c}$ .

$\Rightarrow \vec{a}, \vec{b}, \vec{c}$ , are pairwise perpendicular.

6) If  $\vec{a}, \vec{b}, \vec{c}$  are three vectors of which every pair is non – collinear. If the vectors  $\vec{a} + 2\vec{b}, \vec{b} + 3\vec{c}$  collinear with  $\vec{c}$  and  $\vec{a}$  respectively, then  $\vec{a} + 2\vec{b} + 6\vec{c} =$

- A)  $\vec{a}$     B)  $\vec{b}$     C)  $\vec{c}$     D)  $\vec{0}$

Correct Answer: D

Solution: If  $\vec{a}, \vec{b}, \vec{c}$  are pair wise non collinear then one vector cannot be written as multiple of other, or if  $l\vec{a} + m\vec{b} = \vec{0}$  then  $l = 0, m = 0$ .

$\vec{a} + 2\vec{b}$  is collinear with  $\vec{c}$  then  $\vec{a} + 2\vec{b} = k\vec{c}$  and  $\vec{b} + 3\vec{c}$  is collinear with  $\vec{a}$  then

$$\vec{b} + 3\vec{c} = m\vec{a} \Rightarrow 6\vec{c} - \vec{a} = 2m\vec{a} - k\vec{c} \Rightarrow (k+6)\vec{c} = (2m+1)\vec{a}$$

$$\vec{a} \text{ and } \vec{c} \text{ are non collinear} \Rightarrow k = -6 \Rightarrow \vec{a} + 2\vec{b} + 6\vec{c} = \vec{0}$$

7) If  $\vec{r} = 3\vec{p} + 4\vec{q}$  and  $2\vec{r} = \vec{p} - 3\vec{q}$  then

- A)  $\vec{r}, \vec{q}$  are having same direction and  $|\vec{r}| < 2|\vec{q}|$   
 B)  $\vec{r}, \vec{q}$  are having opposite direction and  $|\vec{r}| > 2|\vec{q}|$   
 C)  $\vec{r}, \vec{q}$  are having opposite direction and  $|\vec{r}| < 2|\vec{q}|$   
 D)  $\vec{r}, \vec{q}$  are having same direction and  $|\vec{r}| > 2|\vec{q}|$

Correct Answer: B

Solution: Solving  $\vec{r} = 3\vec{p} + 4\vec{q}, 6\vec{r} = 3\vec{p} - 9\vec{q}$ .

$$\text{we get } \vec{r} = \frac{-13\vec{q}}{5} \Rightarrow \text{then } \vec{r} \text{ and } \vec{q} \text{ are in opposite in direction } |\vec{r}| = 2.6|\vec{q}| > 2|\vec{q}|$$

8)  $a^2x^2 + 2xy + 9y^2 = 0$  represent a pair of distinct lines than 'a' lies in

A)  $[-\frac{1}{3}, \frac{1}{3}]$     B)  $(-\frac{1}{3}, \frac{1}{3})$     C)  $[-\frac{1}{2}, \frac{1}{2}]$     D)  $(-\frac{1}{2}, \frac{1}{2})$

Correct Answer: B

Solution:  $ax^2 + 2hxy + by^2 = 0$  represent pair of distinct lines then  $h^2 - ab > 0$

Consider the lines  $a^2x^2 + 2xy + 9y^2 = 0$

$$h = 1, a = a^2, b = 9 \Rightarrow 1 - 9a^2 > 0 \Rightarrow 9a^2 - 1 < 0 \Rightarrow a \in (-\frac{1}{3}, \frac{1}{3})$$

9) The approximate value of  $\frac{1}{\sqrt[3]{8.08}} =$

A) 0.49    B) 0.4983    C) 0.048    D) 0.483

Correct Answer: B

Solution: If  $f(x) = \frac{1}{\sqrt[3]{x}}$ ,  $x = 8, \delta x = 0.08$

$$f(x + \delta x) = f(x) + f'(x) \delta x = \frac{1}{2} - \frac{1}{48} \times 0.08 = \frac{1}{2}(1 - 0.0333) = 0.4983$$

10) If  $A^2 = A$  then  $(I + A)^4$

A)  $I + 15A$     B)  $I + 7A$     C)  $I + 8A$     D)  $I + 11A$

Correct Answer: A

Solution:  $(I + A)^4 = I + 4A + 6A^2 + 4A^3 + A^4$

$$A^3 = A^2 = A, A^4 = A^3 = A$$

$$\Rightarrow (I + A)^4 = I + 4A + 6A + 4A + A = I + 15A$$

11) 
$$\begin{vmatrix} 2 & a+b+c+d & ab+cd \\ a+b+c+d & 2(a+b)(c+d) & ab(c+d)+cd(a+b) \\ ab+cd & ab(c+d)+cd(a+b) & 2abcd \end{vmatrix}$$

A)  $abcd$     B) 0    C) 1    D)  $a+b+c+d$

Correct Answer: B

Solution: 
$$\begin{vmatrix} 1 & 1 & 0 \\ c+d & a+b & 0 \\ cd & ab & 0 \end{vmatrix} \begin{vmatrix} 1 & a+b & ab \\ 1 & c+d & cd \\ 0 & 0 & 0 \end{vmatrix} = 0.$$
 Since one of the row of determinant is zero.

12) The velocity  $v$  of a particle moving along a straight line when it is at a distance  $X$  from the point of start is given by  $a + bv^2 = x^2$ , then the acceleration is

A)  $\frac{x}{b}$     B)  $\frac{x}{b^2}$     C)  $\frac{b}{x}$     D)  $\frac{b}{x^2}$

Correct Answer: A

Solution:  $a + bv^2 = x^2$

Differentiating both sides with respect to 't'

$$\Rightarrow 2bv \frac{dv}{dt} = 2x \frac{dx}{dt} = 2xv$$

$$\text{Acceleration} = \frac{dv}{dt} = a = \frac{x}{b}$$

13) If the curves  $\frac{x^2}{a^2} + \frac{y^2}{4} = 1$ ,  $y^3 = 16x$  cut orthogonally then the value of  $a^2$  is

- A)  $\frac{4}{3}$     B)  $\frac{2}{\sqrt{3}}$     C) 1    D)  $\frac{1}{3}$

Correct Answer: A

Solution: Let  $(x_1, y_1)$  be point of intersection of given curves. Slopes of tangents drawn to the curves at  $(x_1, y_1)$ .

$$m_1 = -\frac{4x_1}{a^2 y_1}, m_2 = \frac{16}{3y_1^2} \quad m_1 m_2 = -1$$

$$\Rightarrow 64x_1 = 3a^2 y_1^3 = 3a^2 \times 16x_1$$

$$\Rightarrow a^2 = \frac{4}{3}$$

14)  $f(x) = 3 + 12x - 19x^2 + 2x^3$  is strictly increasing in the interval

- A)  $(\frac{1}{3}, 6)$     B)  $(-\infty, 1) \cup (6, \infty)$     C)  $(1, 3)$     D)  $(-\infty, \frac{1}{3}) \cup (6, \infty)$

Correct Answer: D

$$\text{Solution: } f(x) = 3 + 12x - 19x^2 + 2x^3$$

$f(x)$  is increasing  $\Rightarrow f'(x) > 0$

$$\Rightarrow f'(x) = 12 - 38x + 6x^2 = 6x^2 - 36x - 2x + 12 = 6x(x - 6) - 2(x - 6) = (6x - 2)(x - 6) > 0 \Rightarrow x \in (-\infty, \frac{1}{3}) \cup (6, \infty)$$

15) If the projection of the line segment  $\overline{PQ}$  on the axes are 3, 4, 12 then the length of  $PQ =$

- A) 12    B) 13    C)  $\sqrt{50}$     D)  $2\sqrt{5}$

Correct Answer: B

Solution: If  $P(x_1, y_1, z_1), Q(x_2, y_2, z_2)$  are ends of line segment PQ then Projection of PQ on  $x, y, z$ -axes are  $x_2 - x_1, y_2 - y_1, z_2 - z_1$

$$\text{Length of PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} = \sqrt{3^2 + 4^2 + 12^2} = 13$$

16) If A, B, C are acute angles such that  $\tan A = \frac{1}{2}$ ,  $\tan B = \frac{1}{5}$  and  $\tan C = \frac{1}{8}$  then  $A + B + C =$

- A) 0    B)  $\frac{\pi}{6}$     C)  $\frac{\pi}{4}$     D)  $\frac{\pi}{2}$

Correct Answer: C

$$\text{Solution: } \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{1}{2} + \frac{1}{5}}{1 - \frac{1}{2} \times \frac{1}{5}} = \frac{7}{9}$$

$$\tan(A + B + C) = \frac{\tan(A+B) + \tan C}{1 - \tan(A+B)\tan C} = \frac{\frac{7}{9} + \frac{1}{8}}{1 - \frac{7}{9} \times \frac{1}{8}} = 1 \Rightarrow A + B + C = \frac{\pi}{4}$$

17) Point  $A(2, 1), B(3, 7)$ . C is any point on the line  $3x - 2y = 1$ , then locus of point D such that ABCD is a parallelogram is  $3x - 2y = K$  then  $K =$

- A) 20    B) 18    C) -20    D) -18

Correct Answer: D

Solution: Point  $A(2, 1), B(3, -7), C \dots$  Let the coordinates of C be  $(\alpha, \frac{3\alpha-1}{2})$  and that of D be  $(h, k)$ ,

Then  $2 + \alpha = 3 + h$  and  $\frac{3\alpha-1}{2} + 1 = k - 7$

Therefore  $3 + 3h = 2k - 15$ .

Therefore locus of C is  $3x - 2y + 18 = 0$

18) If the sum of two of the roots of  $x^3 + px^2 + qx + r = 0$  is zero then  $pq =$

- A) -r    B) r    C) 2r    D) -2r

Correct Answer: B

Solution: Let  $\alpha, \beta, \gamma$  be the roots of equation  $x^3 + px^2 + qx + r = 0$ .

Sum of two roots is zero.

Assume that  $\alpha + \beta = 0$

Sum of the roots  $= \alpha + \beta + \gamma = -p \Rightarrow \gamma = -p$

$\gamma$  is a root of an equation  $x^3 + px^2 + qx + r = 0 \Rightarrow (\gamma)^3 + p(\gamma)^2 + q(\gamma) + r = 0$

Substitute -p in place of  $\gamma$  we get  $\Rightarrow (-p)^3 + p(-p)^2 + q(-p) + r = 0$

$\Rightarrow pq = r$

19) The slope of the tangent to the curve  $y = \frac{8}{4+x^2}$  at  $x = 2$  on it is

- A) -2    B) -0.5    C) 0.5    D) 2

Correct Answer: B

Solution:  $\frac{dy}{dx} = \frac{-16x}{(4+x^2)^2}$

$(\frac{dy}{dx})_{x=2} = \frac{32}{64} = -\frac{1}{2}$

Slope of tangent to the curve at  $x = 2$  is  $= -\frac{1}{2}$

20) If  $\tan 40^\circ + 2\tan 10^\circ = \cot x$  then  $x =$

- A)  $75^\circ$     B)  $85^\circ$     C)  $30^\circ$     D)  $40^\circ$

Correct Answer: D

Solution:  $\tan 40^\circ + 2\tan 10^\circ = \frac{\sin 40^\circ}{\cos 40^\circ} + \frac{\sin 10^\circ}{\cos 10^\circ} + \tan 10^\circ$

$= \frac{\sin 40^\circ \cos 10^\circ + \cos 40^\circ \sin 10^\circ}{\cos 40^\circ \cos 10^\circ} + \tan 10^\circ$

$= \frac{1 + \sin 10^\circ}{\cos 10^\circ} = \frac{1 + \cos 80^\circ}{\sin 80^\circ} = \frac{2\cos^2 40^\circ}{2\sin 40^\circ \cos 40^\circ} = \cot 40^\circ$

21) If  $\sinh x = \frac{3}{4}$ , then  $\sinh(2x) =$

- A)  $\frac{5}{8}$     B)  $\frac{15}{8}$     C)  $\frac{7}{8}$     D)  $\frac{17}{8}$

Correct Answer: B

Solution:  $\cosh^2 x - \sinh^2 x = 1$ .

$\sinh x = \frac{3}{4} \Rightarrow \cosh x = \sqrt{1 + \frac{9}{16}} = \frac{5}{4}$

$\sinh 2x = 2 \sinh x \cosh x$

$= 2 \times \frac{3}{4} \times \frac{5}{4} = \frac{15}{8}$

22) In a triangle ABC, if  $C = 60^\circ$  then  $\frac{a}{b+c} + \frac{b}{c+a} =$

- A)2 B)4 C)3 D)1

Correct Answer: D

Solution:  $C = 60^\circ \Rightarrow c^2 = a^2 + b^2 - ab$

$$\frac{a}{b+c} + \frac{b}{c+a} = \frac{ac+a^2+b^2+bc}{bc+ab+c^2+ac} = \frac{a^2+b^2+ac+bc}{bc+ab+ac+a^2+b^2-ab} = \frac{a^2+b^2+ac+bc}{a^2+b^2+ac+bc} = 1$$

23) In a  $\Delta ABC$   $r_1 > r_2 > r_3$ , then its sides are related as

- A)  $a < b < c$  B)  $a < b > c$  C)  $a > b < c$  D)  $a > b > c$

Correct Answer: D

Solution:  $r_1 = \frac{\Delta}{(s-a)}$

$$r_2 = \frac{\Delta}{(s-b)}$$

$$r_3 = \frac{\Delta}{(s-c)}$$

$$r_1 > r_2 > r_3 \Rightarrow \frac{\Delta}{(s-a)} > \frac{\Delta}{(s-b)} > \frac{\Delta}{(s-c)}$$

$$(s-a) < (s-b) < (s-c)$$

$$\Rightarrow -a < -b < -c$$

$$\Rightarrow a > b > c$$

24) The area of the triangle formed by the tangent ,normal at  $(1,\sqrt{3})$  to the circle  $x^2 + y^2 = 4$  and the X- axes is

- A)  $4\sqrt{3}$  B)  $\frac{7}{2}\sqrt{3}$  C)  $2\sqrt{3}$  D)  $\frac{1}{2}\sqrt{3}$

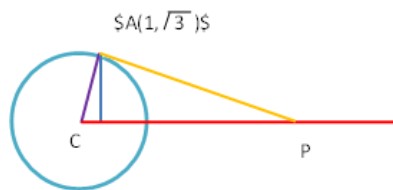
Correct Answer: C

Solution: Equation of tangent to the circle  $x^2 + y^2 = 4$  at  $(1,\sqrt{3})$  is  $S_1 = 0$

i.e  $x(1) + y(\sqrt{3}) = 4$ .....(1)

Equation of x-axis is  $y = 0$ ----(2)

Solving (1) and (2) we get  $(4, 0)$



Centre of the circle  $(0, 0)$

Distance between centre and the point of intersection is 4

Height of the triangle is equal to ordinate of the point  $= \sqrt{3}$

Area of the triangle  $\frac{1}{2} \times 4\sqrt{3} = 2\sqrt{3}$

25) The number of ways of mixed doubles tennis game be arranged from a group of 10 players consisting of 6 men and 4 women is

- A) 48 B) 90 C) 120 D) 180

Correct Answer: D

Solution: 2 men and 2 women are required to play mixed double tennis.

They can be selected from 6 men and 4 women can be selected in  ${}^6C_2, {}^4C_2$  ways.

After selecting 2 men  $M_1, M_2$ , 2 women  $W_1, W_2$  they can interchanged in  $(M_1, W_2), (M_2, W_1) : (M_1, W_1), (M_2, W_2)$ .

Hence total number of ways of selecting for double tennis game  $= {}^6C_2 \times {}^4C_2 \times 2 = 180$

26) The expression  $2x^2 + 4x + 7$  has minimum value "m" at  $x = \alpha$ , then  $(\alpha, m) =$

A)(5, -1)    B)(5, 1)    C)(-1, -5)    D)(-1, 5)

Correct Answer: D

$$\text{Solution: } \alpha = \frac{-b}{2a} = \frac{-4}{4} = -1$$

$$\text{Minimum value} = m = \frac{4ac - b^2}{4a} = \frac{4(2)(7) - 16}{8} = 5$$

point = (-1, 5)

27)  $\alpha, \beta$  are the roots of the equation  $x^2 - px + q = 0$  and  $\alpha, \beta, \gamma$  are the roots of the equation  $x^3 - ax^2 + bx - c = 0$

A)  $p + \gamma = a$     B)  $p\gamma + q = b$     C)  $q\gamma = c$     D) 1, 2, 3 are true

Correct Answer: 4

$$\text{Solution: } \alpha, \beta \text{ are the roots of the equation } x^2 - px + q = 0 \Rightarrow \alpha + \beta = p, \alpha\beta = q$$

$$\alpha, \beta, \gamma \text{ are the roots of the equation } x^3 - ax^2 + bx - c = 0 \Rightarrow \alpha + \beta + \gamma = a \Rightarrow p + \gamma = a$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = b, \alpha\beta\gamma = c$$

$$p\gamma + q = \alpha\gamma + \beta\gamma + \alpha\beta = b \quad q\gamma = \alpha\beta\gamma = c$$

28) The vertices of a triangle are  $(ab, \frac{1}{ab})$   $(bc, \frac{1}{bc})$   $(ca, \frac{1}{ca})$  where a, b, c are the roots of the equation  $x^3 - 3x^2 + 6x + 1 = 0$  then its centroid is

A)(-2, 1)    B)(2, -1)    C)(3, 2)    D)(-3, 2)

Correct Answer: 2

$$\text{Solution: } a, b, c \text{ are the roots of the equation } x^3 - 3x^2 + 6x + 1 = 0$$

$$a + b + c = 3, ab + bc + ca = 6, abc = -1$$

$$\text{Centroid } G = \left( \frac{ab+bc+ca}{3}, \frac{a+b+c}{3abc} \right)$$

$$G = (2, -1)$$

29) The numerically greatest term in the expansion of  $(5x - 6y)^{14}$  when  $x = \frac{2}{5}, y = \frac{1}{2}$  is

A)  ${}^{14}C_6 \cdot 2^8 \cdot 3^6$     B)  ${}^{14}C_6 \cdot 2^6 \cdot 3^8$     C)  ${}^{14}C_5 \cdot 2^6 \cdot 3^8$     D)  ${}^{14}C_7 \cdot 2^8 \cdot 3^6$

Correct Answer: B

Solution: The numerically greatest term in the expansion of  $(5x - 6y)^{14}$  is equivalent to the numerically greatest term in the expansion of  $(5x)^{14} \left(1 - \frac{6y}{5x}\right)^{14}$  which is equivalent to numerically greatest term in the expansion of  $\left(1 - \frac{6y}{5x}\right)^{14}$

Numerically greatest term in the expansion of  $(1 + x)^n$  is  $T_{p+1}$

where  $r = \frac{(n+1)|x|}{1+|x|} = p + f$  where p is integer and f is fractional part.

and  $T_p = T_{p+1}$  when  $r = p$  which is exactly integer.

$$\text{here } x = -\frac{6y}{5x} = -\frac{6 \times \frac{1}{2}}{5 \times \frac{2}{5}} = -\frac{3}{2} = -1.5$$

$$r = \frac{(n+1)|x|}{1+|x|} = \frac{15 \times 1.5}{1+1.5} = 9$$

$\Rightarrow T_9 = T_{10}$  is numerically greatest term.

$$\Rightarrow T_{10} = {}^{14}C_9 (5x)^5 (-6y)^9$$

$$\Rightarrow T_{10} = {}^{14}C_9 (2)^5 (-3)^9$$

$$T_9 = {}^{14}C_8 (2)^6 (-3)^8 = {}^{14}C_6 (2)^6 (-3)^8$$

30) The number of rational terms in the expansion of  $(\sqrt{2} + \sqrt[4]{3})^{100}$  is

- A) 25    B) 26    C) 27    D) 28

Correct Answer: B

Solution: General term  $= T_{r+1} = {}^{100}C_r (\sqrt{2})^{100-r} \times (\sqrt[4]{3})^r$

$(2)^{\frac{100-r}{2}}$  becomes rational if  $r = 0, 2, 4, \dots, 100$

$(3)^{\frac{r}{4}}$  becomes rational if  $r = 0, 4, 8, 12, \dots, 100$

Both becomes rational if  $r = 0, 4, 8, \dots, 100$

Number of terms  $= 25 + 1 = 26$

31) The locus of the point of intersection of the lines  $x = \frac{1-t^2}{1+t^2}$   $y = \frac{2t}{1+t^2}$  where t is a parameter is a circle whose radius is

- A) 2    B) 1    C) 4    D)  $\frac{1}{2}$

Correct Answer: B

Solution: put  $t = \tan\theta$

then  $x = \cos 2\theta$ ,  $y = \sin 2\theta$  and  $x^2 + y^2 = 1$ .

Hence radius = 1.

32) The equation to the locus of the point of intersection of any two perpendicular tangents to  $x^2 + y^2 = 4$  is

- A)  $x^2 + y^2 = 8$     B)  $x^2 + y^2 = 12$     C)  $x^2 + y^2 = 16$     D)  $x^2 + y^2 = 4\sqrt{3}$

Correct Answer: A

Solution: Let point of intersection of tangents is  $(x_1, y_1)$  and angle between tangents is

$$2 \tan^{-1} \left( \frac{r}{\sqrt{s_{11}}} \right) = 90^\circ \Rightarrow \sqrt{s_{11}} = r.$$

$$\Rightarrow \text{Locus of } (x_1, y_1) \text{ is } x^2 + y^2 - 2r^2 = 0 \Rightarrow x^2 + y^2 - 8 = 0$$

33) Nine toys are to be packed in 9 boxes. If 5 of them are too big for 3 boxes, then the number of ways in which they can be packed is

- A)  ${}^6P_5 \cdot 4!$     B)  ${}^6P_5 \cdot 3!$     C)  $6!3!2!$     D)  $5!3!$

Correct Answer: A

Solution: 5 big toys can be placed in remaining 6 boxes in  ${}^6P_5$  ways. remaining 4 toys can be placed in remaining 4 boxes in  $4!$  ways. Hence total number of ways of putting 9 toys in 9 boxes in  ${}^6P_5 \times 4!$  ways.

34) The number of  $3 \times 3$  symmetric matrices using -1, -1, -1, 1, 1, 1, 2, 2, 2 is

- A) 24    B) 36    C) 48    D) 52



Correct Answer: B

Solution: A is said to be symmetric if  $A^T = A$

i.e elements below the diagonal must be same as above the diagonal.

So to decide number of symmetric matrices, its enough to fix diagonal elements and either the elements above the diagonal or below the diagonal.

There are only 3 distinct numbers -1, 1, 2.

These numbers along the diagonal, they can be arranged in 3! ways.

Below the diagonal there are 3 elements, which has to be filled with -1, 1 and 2

This can be done in 3! ways.

Hence number of  $3 \times 3$  matrices =  $3! \times 3! = 36$

35) The number of ways of selections 2 squares on a chess board so as to have a side in common is

- A) 110    B) 111    C) 112    D) 114

Correct Answer: C

Solution: Number of squares having common side = Number of ways of choosing adjacent squares in any row or column. Number of adjacent squares in a row which have commonside = 7. Number of adjacent squares in 8 rows =  $8 \times 7 = 56$ . Similarly number of adjacent squares in 8 columns = 56. Hence total number of squares having adjacent sides = 112.

36)  $P(A \cup B) = \frac{1}{2}$ ;  $P(\bar{A}) = \frac{2}{3} \Rightarrow P(\bar{A} \cap B) =$

- A)  $\frac{1}{3}$     B)  $\frac{1}{4}$     C)  $\frac{1}{5}$     D)  $\frac{1}{6}$

Correct Answer: D

Solution:  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\frac{1}{2} = 1 - P(\bar{A}) + P(B) - P(A \cap B)$$

$$\Rightarrow \frac{1}{2} = \frac{1}{3} + P(B) - P(A \cap B)$$

$$\Rightarrow P(\bar{A} \cap B) = P(B) - P(A \cap B) = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

37) An unbiased coin is tossed  $n$  times. The probability that head will present itself, odd number of times is

- A)  $\frac{1}{4}$     B)  $\frac{1}{3}$     C)  $\frac{1}{2}$     D)  $\frac{1}{5}$

Correct Answer: C

Solution: When coin is tossed  $n$  times then possible number of heads are 0, 1, 2, 3, 4, . . . .  $n$ .

Number of possible ways of getting  $r$  heads =  ${}^n C_r$

Sum of all possible number of ways of getting heads  $\sum_{r=0}^n {}^n C_r = 2^n$

Sum of all possible ways of getting odd number of heads =  ${}^n C_1 + {}^n C_3 + {}^n C_5 + \dots = 2^{n-1}$

Hence probability of getting odd number of heads =  $P(E) = \frac{2^{n-1}}{2^n} = \frac{1}{2}$

38) A linear function that map the set  $\{-2,2\}$  onto the set  $\{0,4\}$

- A)  $(x - 2)$     B)  $(2 - x)$     C)  $2 + x$     D) both B and C

Correct Answer: D

Solution: A linear function  $f(x)$  is in the form of  $ax + b$

$$\text{Case (1)} : f(x) = ax + b, f(-2) = 0, f(2) = 4 \Rightarrow f(x) = 2 + x$$

$$\text{Case (1)} : f(x) = ax + b, f(-2) = 4, f(2) = 0 \Rightarrow f(x) = 2 - x$$

$$39) \int \{1 + \tan x \cdot \tan(x + \alpha)\} dx =$$

- A)  $\cot \alpha \log |\cot(x + \alpha)| + c$     B)  $\cot \alpha \log \left| \frac{\cot(x+\alpha)}{\cos x} \right| + c$     C)  $\cot \alpha \log \left| \frac{\cos x}{\cos(x+\alpha)} \right| + c$   
 D)  $\cot \alpha \log \left| \frac{\sin x}{\cot(x+\alpha)} \right| + c$

Correct Answer: C

$$\text{Solution: } \tan \alpha = \tan(x + \alpha - x) = \frac{\tan(x+\alpha) - \tan x}{1 + \tan(x+\alpha)\tan x} \Rightarrow$$

$$1 + \tan(x + \alpha)(\tan x) = \frac{\tan(x+\alpha) - \tan x}{\tan \alpha} \int 1 + \tan(x + \alpha)(\tan x) dx = \int \frac{\tan(x+\alpha) - \tan x}{\tan \alpha} dx$$

$$= \frac{1}{\tan \alpha} \int (\tan(x + \alpha) - \tan x) dx = \frac{\log \sec(x+\alpha) - \log \sec x}{\tan \alpha} = \cot \alpha \log \left( \frac{\cos x}{\cos(x+\alpha)} \right) + c$$

$$40) \int \tan^7 x dx + \int \tan^9 x dx =$$

- A)  $\frac{\tan^7 x}{7} + c$     B)  $\tan^7 x + c$     C)  $\frac{\tan^{10} x}{10} + c$     D)  $\frac{\tan^8 x}{8} + c$

Correct Answer: D

$$\text{Solution: } \int \tan^7 x dx + \int \tan^9 x dx = \int \tan^7 x (\tan^2 x + 1) dx = \int \tan^7 x \sec^2 x dx$$

$$= \int t^7 dt = \frac{t^8}{8} = \frac{\tan^8 x}{8} + c$$

$$41) \text{ The value at } \int_0^{\frac{\pi}{2}} \frac{e^{\sin x}}{e^{\sin x} + e^{\cos x}} dx \text{ is}$$

- A)  $2\pi$     B)  $\frac{\pi}{2}$     C)  $\pi$     D)  $\frac{\pi}{4}$

Correct Answer: D

$$\text{Solution: } I = \int_0^{\frac{\pi}{2}} \frac{e^{\sin x}}{e^{\sin x} + e^{\cos x}} dx = \int_0^{\frac{\pi}{2}} \frac{e^{\cos x}}{e^{\cos x} + e^{\sin x}} dx$$

$$42) \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \log \left( \frac{2 - \sin \theta}{2 + \sin \theta} \right) d\theta =$$

- A) 0    B) 1    C) 2    D) -1

Correct Answer: A

$$\text{Solution: } f(\theta) = \log \left( \frac{2 - \sin \theta}{2 + \sin \theta} \right)$$

$$f(-\theta) = \log \left( \frac{2 - \sin(-\theta)}{2 + \sin(-\theta)} \right) = \log \left( \frac{2 + \sin \theta}{2 - \sin \theta} \right) = -\log \left( \frac{2 - \sin \theta}{2 + \sin \theta} \right) = -f(\theta). \text{ Hence 'f' is odd function.}$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \log \left( \frac{2 - \sin \theta}{2 + \sin \theta} \right) d\theta = 0.$$

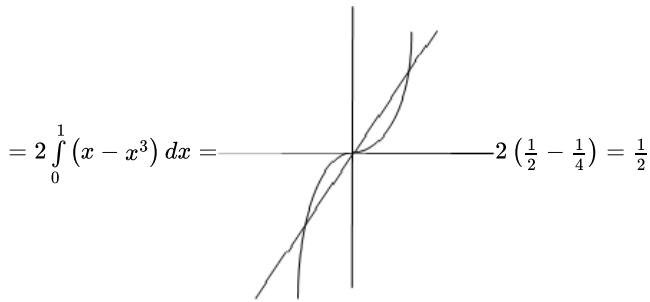
$$43) \text{ The area of the region bounded by } y = x \text{ and } y = x^3 \text{ in square units is}$$

- A) 4    B) 3    C) 2    D)  $\frac{1}{2}$

Correct Answer: D

Solution: Solving  $x = x^3$  gives  $x = 0, x = 1$ .

Area bounded by curves  $y = x$  and  $y = x^3$



44) The order and degree of the differential equation of all tangent lines to the parabola  $x^2 = 4y$  are

- A) 1,2    B) 2,2    C) 3,1    D) 4,1

Correct Answer: A

Solution:  $y = t^2 \Rightarrow x = 2t$

$$\frac{dy}{dx} = 2t \Rightarrow t = \frac{y'}{2}$$

Equation of tangent to the parabola in parametric form is  $yt - x - t^2 = 0$

where t is parameter.

$$\frac{yy'}{2} - x - \frac{y'^2}{4} = 0$$

Order = 1, Degree = 2.

45) The Differential equation whose solution is  $y = ce^{-2x}$  where c is an arbitrary constant is

- A)  $\frac{dy}{dx} - y = 0$     B)  $\frac{dy}{dx} + y = 0$     C)  $\frac{dy}{dx} + 2y = 0$     D)  $\frac{dy}{dx} - 2y = 0$

Correct Answer: C

Solution:  $y = ce^{-2x} \Rightarrow y' = -2ce^{-2x} \Rightarrow y' = -2y$

i.e  $y' + 2y = 0$

$$46) \sum_{m=1}^n \tan^{-1} \left( \frac{2m}{m^4 + m^2 + 2} \right) =$$

- A)  $\tan^{-1} \left( \frac{n^2+n}{n^2+n+2} \right)$     B)  $\tan^{-1} \left( \frac{n^2-n}{n^2-n+2} \right)$     C)  $\tan^{-1} \left( \frac{n^2+n+2}{n^2+n} \right)$     D)  $\frac{\pi}{4}$

Correct Answer: B

$$\text{Solution: } \sum_{m=1}^n \tan^{-1} \left( \frac{2m}{m^4 + m^2 + 2} \right) = \sum_{m=1}^n \tan^{-1} \left( \frac{(m^2+m+1) - (m^2-m+1)}{1 + (m^2+m+1)(m^2-m+1)} \right)$$

$$= \sum_{m=1}^n [\tan^{-1}(m^2 + m + 1) - \tan^{-1}((m^2 - m + 1))]$$

$$= \tan^{-1}(n^2 + n + 1) - \tan^{-1}(1)$$

$$\tan^{-1} \left( \frac{n^2+n}{n^2+n+2} \right)$$

47) The area of the triangle inscribed in the parabola  $y^2 = 4x$  with the vertices, whose ordinates are 1, 2, 4 is

- A)  $\frac{7}{2}$  sq. units    B)  $\frac{5}{2}$  sq. units    C)  $\frac{3}{2}$  sq. units    D)  $\frac{3}{4}$  sq. units

Correct Answer: C

Solution:  $2at = 1$  and  $a = 1 \Rightarrow t = \frac{1}{2} \Rightarrow$  one of the vertex is  $(\frac{1}{4}, 1)$ .

Similarly other vertices are  $(1, 2), (4, 4)$ . The area of the triangle with vertices  $(x_1, y_1), (x_2, y_2)$  and  $(x_3, y_3)$  is

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 - x_2 & x_1 - x_3 \\ y_1 - y_2 & y_1 - y_3 \end{vmatrix}$$

Area =  $(x_1, y_1), (x_2, y_2)$  and  $(x_3, y_3)$  is

$$\Delta = \frac{1}{2} \begin{vmatrix} \frac{1}{4} - 1 & \frac{1}{4} - 4 \\ 1 - 2 & 1 - 4 \end{vmatrix}$$

$$= \frac{1}{2} \left( \frac{9}{4} + \frac{15}{4} \right) = \frac{3}{2}$$

48) If the angle between the lines joining the foci to an extremity of minor axis of an ellipse is  $90^\circ$  its eccentricity is

- A)  $\frac{1}{2}$     B)  $\frac{\sqrt{3}}{2}$     C)  $\frac{1}{\sqrt{3}}$     D)  $\frac{1}{\sqrt{2}}$

Correct Answer: D

Solution:  $\tan 45^\circ = \frac{ae}{b} \Rightarrow b = ae \Rightarrow b^2 = a^2 e^2 \Rightarrow 2e^2 = 1 \Rightarrow e = \frac{1}{\sqrt{2}}$

49)

The product of focal distances of the point  $(4,3)$  on the hyperbola  $x^2 - y^2 = 7$  is

- A) 25    B) 12    C) 9    D) 16

Correct Answer: A

Solution: Eccentricity of rectangular hyperbola =  $\sqrt{2}$ .

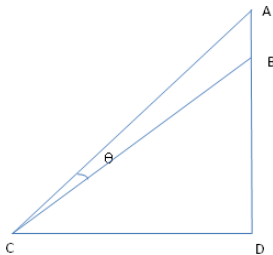
Product of focal distances =  $(ex_1 - a)(ex_1 + a) = 16 \times 2 - 7 = 25$

50) The upper  $\frac{3}{4}$ th portion of a vertical pole subtends an angle  $\tan^{-1}(\frac{3}{5})$  at point in the horizontal plane through its foot and at a distance 40m from the foot. A possible height of the vertical pole is

- A) 20    B) 30    C) 40    D) 60

Correct Answer: C

Solution:



Let AD = Height of vertical pole = h

AB = Upper part of the vertical pole =  $\frac{3h}{4}$

$$\angle ACB = \theta = \tan^{-1} \frac{3}{5}$$

$$\angle BCD = \alpha \Rightarrow \tan \alpha = \frac{h}{40}$$

$$\tan(\alpha + \theta) = \frac{h}{40}$$

$$\Rightarrow \tan(\alpha + \theta) = \frac{\frac{h}{40} + \frac{3}{5}}{1 + \frac{h}{40} \times \frac{3}{5}} = \frac{h}{40}$$

$$\Rightarrow \frac{\frac{h+24}{40}}{\frac{200+3h}{200}} = \frac{h}{40}$$

$$\Rightarrow \frac{5h+120}{200+3h} = \frac{h}{40}$$

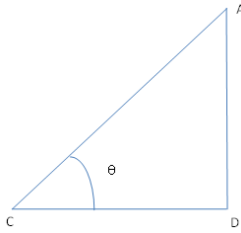
$$\Rightarrow 200h + 4800 = 200h + 3h^2 \Rightarrow h^2 = 1600 \Rightarrow h = \sqrt{1600} = 40$$

51) The angle of elevation of the Sun, when the length of the shadow of a pole is  $\sqrt{3}$  times the height of the pole is

- A)  $30^\circ$    B)  $45^\circ$    C)  $60^\circ$    D)  $90^\circ$

Correct Answer: A

Solution:



AD = Height of the pole = h

CD = Length of the shadow

$$\tan \theta = \frac{h}{\sqrt{3}h} \Rightarrow \theta = \frac{\pi}{6}$$

52)  $\lim_{x \rightarrow 0} \frac{a^{\tan x} - a^{\sin x}}{\tan x - \sin x} = (a > 0)$

- A) 1   B) 0   C)  $\log a$    D) -1

Correct Answer: A

Solution: Given limit can be written as

$$= \lim_{x \rightarrow 0} a^{\sin x} \lim_{x \rightarrow 0} \left( \frac{a^{\tan x - \sin x} - 1}{\tan x - \sin x} \right)$$

$$= \lim_{x \rightarrow 0} a^{\sin x} = a^{\sin 0} = 1$$

Let  $\tan x - \sin x = y$ . As  $x \rightarrow 0$  then  $y \rightarrow 0$

$$\lim_{y \rightarrow 0} \left( \frac{a^y - 1}{y} \right) = 1$$

53) By translating the axes, the equation  $xy - x + 2y = 6$  has changed to  $XY = k$ , then  $k =$

- A) 5   B) 4   C) 3   D) 7

Correct Answer: B

Solution: To remove first degree terms of  $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$  axes has to be translated to the the point

$$\left( \frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2} \right)$$

Comparing the given equation  $xy - x + 2y = 6$  with original equation

$$a = 0, b = 0, 2h = 1, 2g = -1, 2f = 2, c = -6$$

$$\text{Point} = \left( \frac{-f}{h}, \frac{-g}{h} \right) = (-2, 1)$$

$$xy - x + 2y - 6 = 0 \Rightarrow (X - 2)(Y + 1) - (X - 2) + 2(Y + 1) - 6 = 0$$

$$\Rightarrow XY - 2 + 2 + 2 - 6 = 0 \Rightarrow XY = 4$$

(OR)

$$xy - x + 2y - 6 = x(y - 1) + 2(y - 1) = 4 \Rightarrow (x + 2)(y - 1) = 4$$

Let  $x + 2 = X, y - 1 = Y$  We get  $XY = 4$ 54) If  $\sin(x + y) = \log(x + y)$  then  $\frac{dy}{dx} =$ 

- A) 1    B) -1    C) -2    D) 2

Correct Answer: B

Solution: Given equation  $\sin(x + y) = \log(x + y)$ 

Differentiating with respect to x both sides we get

$$\cos(x + y)\left(1 + \frac{dy}{dx}\right) = \left(\frac{1}{x+y}\right)\left(1 + \frac{dy}{dx}\right)$$

$$\Rightarrow \left(1 + \frac{dy}{dx}\right)\left(\cos(x + y) - \frac{1}{x+y}\right) = 0$$

$$\Rightarrow \left(1 + \frac{dy}{dx}\right) = 0 \Rightarrow \frac{dy}{dx} = -1$$

55) If  $f(x) = \begin{cases} [x] & \text{if } -3 < x \leq -1 \\ |x| & \text{if } -1 < x < 1 \\ [x] & \text{if } 1 \leq x < 3 \end{cases}$  then  $f$  is continuous on

- A) (-3,3)    B) [-1,1]    C) (-3,1)    D) (-1,1]

Correct Answer: D

Solution:  $|x|$  is continuous function on  $(-1,1)$  $[x]$  is discontinuous at integers.Hence  $[x]$  is discontinuous at  $x = -2$  over the interval  $(-3, -1)$  $[x]$  is discontinuous at  $x = 2$  over the interval  $(1,3)$ At  $x = 1$ 

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} |[x]| = 1$$

$$\lim_{x \rightarrow 1^-} |x| = 1$$

$$f(1) = 1$$

Hence  $f$  is continuous at  $x = 1$ At  $x = -1$ 

$$\lim_{x \rightarrow -1^+} f(x) = \lim_{x \rightarrow -1^+} |x| = 1$$

$$\lim_{x \rightarrow -1^-} [x] = -2$$

Hence  $f$  is discontinuous at  $x = -1$ Hence  $f$  is continuous over  $(-1,1]$ 56) If  $X$  is a binomial variate with  $n = 6$  and  $P(X = 2) = 4P(X = 4)$ , then the parameter  $p$  of  $X$  is :

- A)
- $\frac{1}{2}$
- B)
- $\frac{1}{3}$
- C)
- $\frac{2}{3}$
- D)
- $\frac{3}{4}$

Correct Answer: B

Solution:  $P(X = x) = {}^n C_x p^x q^{n-x}$ 

$$P(X = 2) = {}^6 C_2 p^2 q^4$$

$$P(X = 4) = {}^6 C_4 p^4 q^2$$

Given  $P(X = 2) = 4P(X = 4)$

$${}^6C_2 p^2 q^4 = 4({}^6C_4 p^4 q^2) \Rightarrow q^2 = 4p^2 \Rightarrow q = 2p \Rightarrow q + p = 2p + p = 1 \Rightarrow p = \frac{1}{3}$$

57) The minimum value of  $f(x) = |3 - x| + |2 + x| + |5 - x|$  is

- A) 0    B) 7    C) 8    D) 10

Correct Answer: B

Solution: Minimum value of sum of absolute value is obtained at median

$$|3 - x| + |2 + x| + |5 - x|$$

Median of the numbers -2, 3, 5 is 3.

$$\text{Minimum value of the expression } |3 - x| + |2 + x| + |5 - x| = |3 - 3| + |2 + 3| + |5 - 3| = 7$$

58) If  $x = e^{y + e^{y + \dots \text{to } \infty}}$ ,  $x > 0$  then  $\frac{dy}{dx} =$

- A)  $\frac{x}{1+x}$     B)  $\frac{1+x}{x}$     C)  $\frac{1-x}{x}$     D)  $\frac{1}{x}$

Correct Answer: C

Solution:  $x = e^{y + e^{y + \dots \text{to } \infty}}$ ,  $x > 0$

$$\Rightarrow x = e^{y+x}$$

Taking logarithms both sides we get

$$\log x = (x + y)$$

Differentiating both sides with respect to x we get

$$\frac{1}{x} = 1 + \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1-x}{x}$$

59) The mean weight of 150 students in a certain class is 60 kilograms. The mean weight of boys in the class is 70 kilograms and that of the girls is 55 kilograms, then the number of boys and girls are

- A) 100,50    B) 50,100    C) 75,75    D) 60,90

Correct Answer: B

$$\text{Solution: } (b \times 70) + (g \times 55) = (b + g) 60 \Rightarrow b : g = 1 : 2$$

60) If  $y = \log_2 (\log_2^x)$  then  $\frac{dy}{dx} =$

- A)  $\frac{1}{x(\log_2^e)^2}$     B)  $\frac{1}{\log_2 (2x)^2}$     C)  $\frac{(\log_2^e)^2}{x \log_2^e}$     D)  $\frac{(\log_2^e)^2}{x \log_2^e}$

Correct Answer: C

$$\text{Solution: } \frac{d}{dx} (\log_e x) = \frac{1}{x}$$

$$\frac{d}{dx} (\log_2 x) = \frac{d}{dx} \frac{\log_e x}{\log_e 2} = \frac{1}{\log_e 2} \frac{d}{dx} (\log_e x) = \frac{\log_2 e}{x}$$

$$\frac{d}{dx} (\log_2 (\log_2 x)) = \frac{d}{dx} (\log_2 e) (\log_e (\log_2 x)) = (\log_2 e) \frac{d}{dx} (\log_e (\log_2 x))$$

$$= (\log_2 e) \frac{1}{\log_2 x} \frac{d}{dx} (\log_2 x) = (\log_2 e) \frac{1}{\log_2 x} \frac{\log_2 e}{x} = \frac{(\log_2^e)^2}{x \log_2^e}$$

$$61) A = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{pmatrix}, \text{adj } A = \begin{pmatrix} 1 & -3 & 4 \\ -3 & 1 & 0 \\ 1 & 1 & k \end{pmatrix}$$

A) 1    B) 2    C) -4    D) -8

Correct Answer: C

Solution: By definition of adjoint matrix, it is obtained by cofactors of a given matrix .

$$k = \text{Cofactor of } 2 = +(2 - 6) = -4$$

$$62) \begin{vmatrix} x & x+y & x+y+z \\ 2x & 3x+2y & 4x+3y+2z \\ 3x & 6x+3y & 10x+6y+3z \end{vmatrix} = 64 \text{ then } x =$$

A) 2    B) 4    C) 6    D) 8

Correct Answer: B

$$\text{Solution: } \begin{vmatrix} x & x+y & x+y+z \\ 2x & 3x+2y & 4x+3y+2z \\ 3x & 6x+3y & 10x+6y+3z \end{vmatrix} = 64$$

$$R_2 \rightarrow R_2 - 2R_1, R_3 \rightarrow R_3 - 3R_1$$

$$\begin{vmatrix} x & x+y & x+y+z \\ 0 & x & 2x+y \\ 0 & 3x & 7x+3y \end{vmatrix} = 64$$

Expanding determinant with respect to first column we get

$$x \begin{vmatrix} x & 2x+y \\ 3x & 7x+3y \end{vmatrix} = 64$$

$$\Rightarrow x(7x^2 + 3xy - 6x^2 - 3xy) = x^3 = 64 \Rightarrow x = 4$$

63)  $f(x) = \sqrt{25 - 9x^2}$  is strictly decreasing in

A)  $(-\frac{5}{3}, 0)$     B)  $(0, \frac{5}{3})$     C)  $(-\frac{5}{3}, \frac{5}{3})$     D) None

Correct Answer: B

Solution:  $f(x)$  is strictly decreasing then  $f'(x) < 0$

$$f(x) = \sqrt{25 - 9x^2} \Rightarrow f'(x) = \frac{-9x}{\sqrt{25 - 9x^2}} < 0$$

$$\frac{1}{\sqrt{25 - 9x^2}} \text{ is defined if } 25 - 9x^2 > 0 \Rightarrow x \in (-\frac{5}{3}, \frac{5}{3})$$

$$f(x) = \sqrt{25 - 9x^2} > 0 \text{ and } f'(x) < 0 \Rightarrow x > 0$$

$$\Rightarrow x \in (0, \frac{5}{3})$$

64) Stationary point of  $y = x^2 + \frac{250}{x}$  is

A) (1, 5)    B) (5, 1)    C) (5, 25)    D) (5, 75)

Correct Answer: D

Solution: A point  $(a, f(a))$  is stationary point if  $f'(a) = 0$

$$f(x) = x^2 + \frac{250}{x} \Rightarrow f'(x) = 2x - \frac{250}{x^2} = 0 \Rightarrow x = 5$$

$$\text{Stationary value} = f(5) = 25 + 50 = 75$$

$$\text{Stationary point } (5, 75)$$

65) The eccentricity of a rectangular hyperbola is



- A) 1    B)  $\frac{2}{3}$     C)  $\frac{3}{2}$     D)  $\sqrt{2}$

Correct Answer: D

Solution: Equation of rectangular hyperbola  $x^2 - y^2 = a^2$

$$a^2 = a^2(e^2 - 1) \Rightarrow e^2 - 1 = 1 \Rightarrow e = \sqrt{2}$$

66) If it rains a dealer in rain coats can earn Rs. 500/- a day. If it is fair, he can loose Rs. 40/- per day. What is his mean profit if the probability of a fair day is 0.6?

- A) 170    B) 172    C) 274    D) 176

Correct Answer: D

Solution: Random variable X is defined as profit on business

Probability that the day is not fair  $P(X = 500) = 1 - 0.6 = 0.4$

Probability that the day is fair  $P(X = -40) = 0.6 = 0.6$

Mean of the profit =  $\sum xP(X = x) = 500 \times 0.4 - 40 \times 0.6 = 200 - 24 = 176$

67) If  $f : R \rightarrow R$  defined by  $f(x) = x^2 - 10x + 21$  then  $f^{-1}(-3) =$

- A)  $\{-4, 6\}$     B)  $\{2, 4\}$     C)  $\{-4, 4, 6\}$     D)  $\{4, 6\}$

Correct Answer: D

Solution: Let  $f^{-1}(-3) = x \Rightarrow f(x) = -3 \Rightarrow x^2 - 10x + 21 = -3 \Rightarrow x^2 - 10x + 24 = 0 \Rightarrow (x - 4)(x - 6) = 0 \Rightarrow x = 4, x = 6$

68) If there is an error 0.01 cm in the diameter of a sphere when its radius is 5 cm. the percentage error in its surface area = .....

- A) 0.2    B) 0.6    C) 0.5    D)  $\frac{\pi}{10}$

Correct Answer: A

Solution: Surface area of sphere =  $4\pi r^2 = \pi d^2 = f(d)$  where r = radius of sphere, d = diameter of sphere.  $d = 2r$

Percentage error in the diameter of the sphere

$$= \frac{\delta d}{d} \times 100 \simeq \frac{f'(d)}{f(d)} \times 100 = \frac{2\pi d \delta d}{\pi d^2} \times 100 = \frac{2\delta d}{d} \times 100 = \frac{2 \times 0.01}{10} \times 100 = 0.2$$

$$69) \int_0^1 \log\left(\frac{1}{x} - 1\right) dx =$$

- A)  $\frac{1}{2} \log \frac{1}{2}$     B)  $\frac{1}{2} \log 2$     C) 0    D)  $\log \frac{1}{2}$

Correct Answer: C

Solution:  $\int_0^a f(x) dx = \int_0^a f(a-x) dx =$

$$\int_0^1 \log\left(\frac{1}{x} - 1\right) dx = I = \int_0^1 \log \frac{1-x}{x} dx$$

$$= \int_0^1 \log \frac{1-(1-x)}{1-x} dx = \int_0^1 \log \frac{x}{1-x} dx = I$$

$$\text{Adding both the integrals } 2I = \int_0^1 \log \frac{1-x}{x} \times \frac{x}{1-x} dx = \int_0^1 \log 1 dx = 0$$

70) The number of rational terms in the expansion of  $(\sqrt{2} + \sqrt[4]{3})^{100}$  is

- A) 28    B) 26    C) 27    D) 28

Correct Answer: B

Solution: The general term of the expansion  $(\sqrt{2} + \sqrt[4]{3})^{100}$  is  $T_{r+1} = {}^{100}C_r (\sqrt{2})^{100-r} 3^{\frac{1}{4}r}$   
 $= {}^{100}C_r (2)^{\frac{100-r}{2}} 3^{\frac{r}{4}}$

Terms becomes rational if  $r = 0, 4, 8, \dots, 100$

Number of rational terms = 26

71) The locus of a point represented by equations  $x = \frac{a}{2}(t + \frac{1}{t}), y = \frac{a}{2}(t - \frac{1}{t})$

- A)  $x^2 + y^2 = a^2$     B)  $x^2 - y^2 = a^2$     C)  $2x^2 - y^2 = a^2$     D)  $x^2 - 2y^2 = a^2$

Correct Answer: B

Solution:  $x = \frac{a}{2}(t + \frac{1}{t}), y = \frac{a}{2}(t - \frac{1}{t})$

$$\Rightarrow (t + \frac{1}{t}) = \frac{2x}{a}, (t - \frac{1}{t}) = \frac{2y}{a}$$

$$(t + \frac{1}{t})^2 - (t - \frac{1}{t})^2 = 4$$

$$\Rightarrow (\frac{x^2}{a^2} - \frac{y^2}{a^2}) = 4$$

$$\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{a^2} = 1 \Rightarrow x^2 - y^2 = a^2$$

72) The number of partial fractions of  $\frac{2}{x^4+x^2+1}$  is

- A) 2    B) 3    C) 4    D) 5

Correct Answer: A

Solution:  $\frac{2}{x^4+x^2+1} = \frac{2}{x^4+2x^2+1-x^2} = \frac{2}{(x^2+1)^2-x^2} = \frac{Ax+B}{x^2+x+1} + \frac{Cx+D}{x^2-x+1}$

Hence number of partial fractions are = 2

73) In the expansion of  $(1+x)^{43}$  if the coefficients of the  $(2r+1)^{th}$  and  $(r+2)^{th}$  terms are equal the value of 'r' is

- A) 12    B) 13    C) 14    D) 15

Correct Answer: C

Solution: In the expansion of  $(1+x)^{43}$  if the coefficients of the  $(2r+1)^{th}$  and  $(r+2)^{th}$  terms are equal

Coefficients of  $T_{2r+1} = T_{r+2}$  are same

$${}^{43}C_{2r} = {}^{43}C_{r+1}$$

$${}^nC_r = {}^nC_s \Rightarrow r = s \text{ or } n = r + s$$

$${}^{43}C_{2r} = {}^{43}C_{r+1} \Rightarrow 3r + 1 = 43 \Rightarrow r = 14$$

value of 'r' = 14

74)  $\frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ} =$

- A)  $\tan 62^\circ$     B)  $\tan 79^\circ$     C)  $\tan 56^\circ$     D)  $\tan 9^\circ$

Correct Answer: C

$$\text{Solution: } \frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ}$$

Dividing both numerator and denominator with  $\cos 11^\circ$

$$= \frac{1 + \frac{\sin 11^\circ}{\cos 11^\circ}}{1 - \frac{\sin 11^\circ}{\cos 11^\circ}} = \frac{1 + \tan 11^\circ}{1 - \tan 11^\circ}$$

$$= \frac{\tan 45^\circ + \tan 11^\circ}{1 - \tan 45^\circ \tan 11^\circ}$$

$$= \tan(45^\circ + 11^\circ) = \tan 56^\circ$$

75) If the projections of the line segment AB on the yz-plane, zx-plane, xy-plane are  $\sqrt{160}$ ,  $\sqrt{153}$ , 5 respectively then the projection of AB on the z-axis is

- A) 10    B) 11    C) 12    D) 15

Correct Answer: C

Solution: Projection of AB on the YZ, ZX and XY Planes are  $d_1, d_2, d_3$  respectively.

$$\text{Projection of AB on XY plane} = \sqrt{AB^2 - (\text{Projection of AB on normal of the XY plane})^2}$$

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 - (z_2 - z_1)^2}$$

$$\Rightarrow d_1 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Let projection of AB on x-axis, y-axis and z-axis be respectively

$$|x_2 - x_1| = p, |y_2 - y_1| = q, |z_2 - z_1| = r$$

$$d_1 = \sqrt{160}, d_2 = \sqrt{153}, d_3 = 5$$

$$d_1^2 = q^2 + r^2 = 160, d_2^2 = r^2 + p^2 = 153$$

$$d_3^2 = p^2 + q^2 = 25$$

$$p^2 + q^2 + r^2 = \frac{d_1^2 + d_2^2 + d_3^2}{2}$$

$$25 + r^2 = \frac{160 + 153 + 25}{2} = 169$$

$$\therefore r^2 = 144 \Rightarrow r = 12$$

76) The area of the parallelogram formed by the lines

$$4y - 3x - a = 0, 3y - 4x + a = 0, 4y - 3x - 3a = 0, 3y - 4x + 2a = 0 \text{ is}$$

- A)  $\frac{a^2}{5}$     B)  $\frac{a^2}{7}$     C)  $\frac{2a^2}{7}$     D)  $\frac{2a^2}{9}$

Correct Answer: C

Solution: Area of quadrilateral formed by pair of lines

$$a_1x + b_1y + c_1 = 0, a_2x + b_2y + c_2 = 0, a_1x + b_1y + d_1 = 0, a_2x + b_2y + d_2 = 0 = \left| \frac{(c_1 - d_1)(c_2 - d_2)}{a_1b_2 - a_2b_1} \right|$$

$$\text{Given lines are } 3x - 4y + a = 0, 4x - 3y - a = 0, 3x - 4y + 3a = 0, 4x - 3y - 2a = 0$$

$$\text{Area is } \left| \frac{(2a)(-a)}{-16+9} \right| = \frac{2a^2}{7}$$

77) The number  $a^n = 6^n - 5n$  for  $n = 1, 2, 3, \dots$  when divided by 25 leave the remainder is

- A) 9    B) 7    C) 3    D) 1

Correct Answer: D

Solution:

$$a^n = 6^n - 5n = (5 + 1)^n - 5n = 5^n + {}^nC_1 5^{n-1} + \dots + {}^nC_n - 2(5^2) + {}^nC_{n-1} 5 + 5n + 1 - 5n = 25M + 1$$

Hence  $a^n = 6^n - 5n$  divided by 25 gives us remainder 1.

Substitute  $n = 1, 2$  and check it

78)

If the period of  $\sin(x + 8x + 27x + \dots + n^3x)$  is  $\frac{k\pi}{n^2(n+1)^2}$

then k =

- A) 1    B) 2    C) 6    D) 8

Correct Answer: D

Solution:  $\sin(x + 8x + 27x + \dots + n^3x) = \sin(1^3 + 2^3 + \dots + n^3)x = \sin(\Sigma n^3)x$

Period of  $\sin kx = \frac{2\pi}{|k|}$

Period of  $\sin(\Sigma n^3)x = \frac{2\pi}{(\Sigma n^3)} = \frac{8\pi}{n^2(n+1)^2}$

79)

If  $\cos x + \cos y = \frac{1}{3}$ ,  $\sin x + \sin y = \frac{1}{4}$  then  $\sin(x + y) =$

- A)  $\frac{7}{25}$     B)  $\frac{24}{25}$     C)  $\frac{25}{24}$     D)  $\frac{25}{7}$

Correct Answer: B

Solution:  $\cos x + \cos y = \frac{1}{3} \Rightarrow 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = \frac{1}{3} \dots (1)$

$\sin x + \sin y = \frac{1}{4} \Rightarrow 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = \frac{1}{4} \dots (2)$

Dividing (2) with (1) we get  $\tan\left(\frac{x+y}{2}\right) = \frac{4}{3}$

$\sin(x + y) = \frac{2\tan\left(\frac{x+y}{2}\right)}{1+\tan^2\left(\frac{x+y}{2}\right)} = \frac{\frac{8}{3}}{1+\frac{16}{9}} = \frac{24}{25}$

80) The vector component of  $\vec{b}$  perpendicular to  $\vec{a}$  is

- A)  $(\vec{b} \cdot \vec{c})\vec{a}$     B)  $\frac{\vec{a} \times (\vec{b} \times \vec{a})}{|\vec{a}|^2}$     C)  $\vec{a} \times (\vec{b} \times \vec{a})$     D)  $(\vec{b} \times \vec{a})$

Correct Answer: B

Solution: Vector component of  $\vec{b}$  perpendicular to  $\vec{a} = \vec{b} - \frac{(\vec{b} \cdot \vec{a})\vec{a}}{|\vec{a}|^2}$   
 $= \frac{\vec{b}(\vec{a} \cdot \vec{a}) - (\vec{b} \cdot \vec{a})\vec{a}}{|\vec{a}|^2} = \frac{\vec{a} \times (\vec{b} \times \vec{a})}{|\vec{a}|^2}$

81) When a ball is thrown up vertically with velocity  $V_0$ , it reaches a maximum height of 'h'. If one wishes to triple the maximum height then the ball should be thrown with velocity

- A)  $\sqrt{3}V_0$     B)  $3V_0$     C)  $9V_0$     D)  $\frac{3}{2}V_0$

Correct Answer: A

Solution: Maximum height reached by body when it is vertically projected with a velocity  $V_0$  is  $h = \frac{V_0^2}{2g}$

Let V be the velocity to be the required velocity to reach maximum height 3h

$3h = \frac{V^2}{2g} \Rightarrow \frac{3V_0^2}{2g} = \frac{V^2}{2g} \Rightarrow V = \sqrt{3}V_0$

82) From the top of a building 9.8 m high, a ball is thrown horizontally which hits the ground at a distance. The line joining the top of a building to the point where the ball hits the ground makes an angle of  $45^\circ$  with the ground, the initial velocity of projection of the ball is

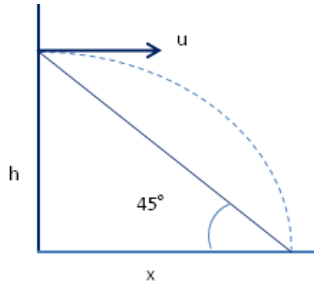
- A)  $4.9 \text{ m s}^{-1}$     B)  $4.9\sqrt{2} \text{ m s}^{-1}$     C)  $9.8 \text{ m s}^{-1}$     D)  $9.8\sqrt{2} \text{ m s}^{-1}$

Correct Answer: B

Solution: The ball is projected horizontally with a velocity  $v$ , then its initial velocity in a vertical direction is zero.

Let the time taken by the ball to reach the ground be ' $t$ '.

i.e. in ' $t$ ' seconds, it travels vertically 9.8m. Assume that it travels distance ' $x$ 'm in a horizontal direction.



$$\Rightarrow h = 9.8m = \frac{gt^2}{2} = \frac{9.8t^2}{2} \rightarrow t = \sqrt{2}sec$$

$$\text{Horizontal distance } x = ut = \sqrt{2}u$$

Since the angle made by the line joining point of projection and the point where it hits the ground is  $45^\circ$

$$\tan 45^\circ = \frac{h}{x} \Rightarrow x = 9.8m$$

$$\Rightarrow u = \frac{9.8}{\sqrt{2}} = 4.9\sqrt{2}$$

83) A force acts for 0.5s on a body of mass 1.5kg initially at rest. When the force ceases to act, the body is found to cover a distance of 5m in 2s. The magnitude of the applied force is

- A) 5.0 N    B) 10 N    C) 12.5 N    D) 7.5 N

Correct Answer: D

Solution: The body moves for 0.5 sec in presence of force, when the force ceases

$$F = 0; a = 0 \Rightarrow v = \text{constant}$$

$$v = \frac{s}{t} = \frac{5}{2} = 2.5 \text{ m/s.}$$

This is the velocity attained by the body when the force ceases.

The acceleration in first 0.5 sec is

$$a = \frac{v-u}{t} = \frac{2.5-0}{0.5} = 5 \text{ m/s}^2$$

$$\text{Magnitude of the force acts on the body} = F = ma = 1.5 \times 5 = 7.5 \text{ N.}$$

84) An electric pump is used to fill an overhead tank of capacity  $9 \text{ m}^3$  kept at a height 10 m above the ground. If the pump takes 5 minutes to fill the tank by consuming  $10 \text{ kW}$  power, the efficiency of the pump should be where  $g = 10 \text{ m/s}^2$

- A) 60%    B) 40%    C) 20%    D) 30%

Correct Answer: D

Solution: The mass of water filled in the tank is given as  $m = dV$

$$\text{Where } d = \text{density of water} = 1000 \text{ kg/m}^3$$

$$V = \text{Volume of the water } V = 9 \text{ m}^3$$

$$\text{Mass} = m = Vd = 1000 \times 9 = 9000 \text{ Kg}$$

$$\text{The raise in the potential energy of the water } E = mgh = 9 \times 10^3 \times 10 \times 10 = 9 \times 10^5$$

$$\text{Observed power of the pump } P = \frac{E}{t} = \frac{mgh}{t} = \frac{9 \times 10^3 \times 10 \times 10}{5 \times 60} = 3 \times 10^3 \text{ W}$$

$$\text{Original power} = 10 \text{ kW} = 10 \times 10^3 \text{ W}$$

$$\text{Efficiency} = \frac{\text{Observed power}}{\text{Original power}} \times 100 = \frac{3 \times 10^3}{10 \times 10^3} \times 100 = 30\%$$

85) A ball is dropped from a height 'h' on a floor of the coefficient to restitution 'e'. The total time covered by the ball just before the second hit is

- A)  $e^2 \sqrt{\frac{2h}{g}}$     B)  $(1 + 2e) \sqrt{\frac{2h}{g}}$     C)  $2e \sqrt{\frac{2h}{g}}$     D)  $\frac{e}{2} \sqrt{h}$

Correct Answer: B

Solution: A ball of mass, 'm' dropped from a height 'h' and its velocity just before contact be  $v_1$

Initially, it has potential energy, the total potential is converted to kinetic energy just before contact.

$$\Rightarrow mgh = \frac{mv_1^2}{2} \Rightarrow v_1^2 = 2gh$$

Let velocity of ball after collision be  $v_2$  and it raises to a height  $h_1$

$$e = \text{coefficient of restitution} = \frac{\text{Relative velocity after collision}}{\text{Relative velocity before collision}} = \frac{v_2}{v_1}$$

$$\text{At highest point } mgh_1 = \frac{mv_2^2}{2} \Rightarrow v_2^2 = 2gh_1$$

$$e = \sqrt{\frac{h_1}{h}} \Rightarrow h_1 = e^2 h$$

(or) you remember  $h_n = e^2 h^n$  problem can be solved directly

$$t = t_1 + 2t_2 = \sqrt{\frac{2h}{g}} + 2\sqrt{\frac{2h_1}{g}}$$

$$= \sqrt{\frac{2h}{g}} + 2e\sqrt{\frac{2h}{g}}$$

$$= (1 + 2e) \sqrt{\frac{2h}{g}}$$

86) Three bodies of masses 1kg, 2kg, 3kg are acted upon by forces  $(i + 2j)$ ,  $(2j + 3k)$  and  $(i - k)$  newton respectively. The magnitude of acceleration of centre of mass is

- A)  $\sqrt{3}$     B)  $2\sqrt{3}$     C)  $\sqrt{\frac{2}{3}}$     D)  $\frac{2}{\sqrt{3}}$

Correct Answer: C

$$\text{Solution: Acceleration of centre of mass } \bar{a}_{cm} = \frac{m_1 \bar{a}_1 + m_2 \bar{a}_2 + m_3 \bar{a}_3}{m_1 + m_2 + m_3} = \frac{\bar{F}_1 + \bar{F}_2 + \bar{F}_3}{m_1 + m_2 + m_3}$$

$$= \frac{i + 2j + 2j + 3k + i - k}{1 + 2 + 3} = \frac{2i + 4j + 2k}{6} = \frac{i + 2j + k}{3}$$

$$\text{Magnitude of acceleration of centre of mass} = |\bar{a}_{cm}| = \frac{1}{3} \times \sqrt{1 + 4 + 1} = \frac{\sqrt{6}}{3} = \sqrt{\frac{2}{3}}$$

87) A block released from rest from the top of a smooth inclined plane of inclination  $45^\circ$  takes  $t$  seconds to reach the bottom. The same block released from rest from the top of a rough inclined plane of the same inclination of  $45^\circ$  takes  $2t$  seconds to reach the bottom. The coefficient of friction is

- A)  $\sqrt{0.5}$     B)  $\sqrt{0.75}$     C) 0.5    D) 0.75

Correct Answer: D

$$\text{Solution: Time taken by the block to reach bottom of smooth inclined plane } t_s = \sqrt{\frac{2h}{g \sin \theta}}$$

$$\text{Time taken by the block to reach bottom of rough inclined plane } t_r = \sqrt{\frac{2h}{g(\sin \theta - \mu_k \cos \theta)}}$$

$$\text{Given : } t_r = 2t_s$$

$$\Rightarrow 2t = \sqrt{\frac{2h}{g(\sin \theta - \mu_k \cos \theta)}}$$

$$\Rightarrow 2\sqrt{\frac{2h}{g \sin \theta}} = \sqrt{\frac{2h}{g(\sin \theta - \mu_k \cos \theta)}}$$

$$\Rightarrow 4 \times \frac{2h}{g \sin \theta} = \frac{2h}{g(\sin \theta - \mu_k \cos \theta)}$$

$$\Rightarrow 4(\sin \theta - \mu_k \cos \theta) = \sin \theta$$

$$4 \times \frac{1}{\sqrt{2}} - \frac{\mu_k}{\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow 1 - \mu_k = \frac{1}{4}$$

$$\Rightarrow \mu_k = 1 - \frac{1}{4} = \frac{3}{4} = 0.75$$

88) If the earth were suddenly shrunk to  $\frac{1}{n}$  of its present radius without any change in its mass, the duration of the new day will be

- A)  $\frac{24}{n} hrs$     B)  $\frac{24}{n^2} hrs$     C)  $24 n^2 hrs$     D)  $\frac{24}{\sqrt{n}} hrs$

Correct Answer: B

Solution: Conservation of angular momentum dictates that the angular momentum before and after the contraction must be the same.

$$I_1 \omega_1 = I_2 \omega_2$$

$$\text{Moment of inertia of earth} = \frac{2Mr_1^2}{5}$$

Where  $r_1$  be the radius of earth before contraction, M is mass of the earth.

$$\frac{2}{5} Mr_1^2 \times \frac{2\pi}{T_1} = \frac{2}{5} Mr_2^2 \times \frac{2\pi}{T_2}$$

$$\Rightarrow \frac{r_2^2}{T_1} = \frac{r_1^2}{T_2}$$

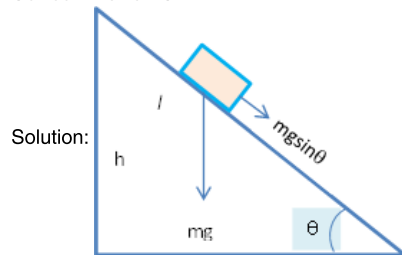
$$\text{Given new radius } r_2 = \frac{r_1}{n}$$

$$T_2 = \frac{T_1}{n^2} = \frac{24}{n^2} hrs$$

89) Two identical hollow spheres roll down two inclined planes of the same height but of different angles of inclination. Then they reach the bottom.

- A) With same speed and in same time    B) With different speeds and in different time  
C) With same speed but in different times    D) With different speeds in same time

Correct Answer: C



Velocity at the bottom of the inclined plane of inclination of angle  $\theta$  when it is released from the top of height  $h$ .

$$\text{Let } \ell \text{ be the length of inclined plane } \frac{h}{\ell} = \sin \theta$$

$$\text{Acceleration } a = g \sin \theta$$

$$v^2 - u^2 = 2a\ell \Rightarrow v^2 - 0 = 2g \sin \theta \ell$$

$$\Rightarrow v = \sqrt{2gh}$$

$$t = \sqrt{\frac{2h}{g \sin \theta}}$$

As 'h' is same  $v$  is same as  $\theta$  is different 't' is different

90) A stretched wire of some length under tension is vibrating with its fundamental frequency. Its length is decreased by 45% and tension is increased by 21%. Now its fundamental frequency.

- A) Increases by 50%    B) Increases by 100%    C) Decreases by 50%    D) Decreases by 25%

Correct Answer: B

$$\text{Solution: Fundamental frequency of stretched wire} = n = \frac{1}{2l} \sqrt{\frac{T}{m}}$$

$$\Rightarrow n \propto \frac{\sqrt{T}}{l}$$

$$\frac{n_2}{n_1} = \sqrt{\frac{T_2}{T_1}} \times \frac{l_1}{l_2}$$

Length decreased by 45%

$$\text{New length } l_2 = l_1 - \frac{45l_1}{100} = \frac{55l_1}{100}$$

Tension increased by 21

$$\text{New tension } T_2 = T_1 + \frac{21T_1}{100} = \frac{121T_1}{100}$$

$$\frac{n_2}{n_1} = \sqrt{\frac{121}{100}} \times \frac{100}{55}$$

$$= \frac{11}{10} \times \frac{100}{55} = \frac{10}{5} = 2$$

$$\frac{n_2}{n_1} - 1 = 1$$

$$\text{Increase in fundamental frequency} = \frac{n_2 - n_1}{n_1} \times 100 = 100\%$$

91) A prism is made up of material of refractive index  $\sqrt{3}$ . The angle of the prism is A. If the angle of minimum deviation is equal to the angle of the prism, then the value of A is

- A)  $30^\circ$     B)  $45^\circ$     C)  $60^\circ$     D)  $75^\circ$

Correct Answer: C

$$\text{Solution: Refractive index of prism } \mu = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

Where A = Angle of the prism, D = Angle of minimum deviation.

Given A = D.

$$\sqrt{3} = \frac{\sin A}{\sin\left(\frac{A}{2}\right)}$$

$$= \frac{\sin\frac{A}{2} \cos\frac{A}{2}}{\sin\frac{A}{2}}$$

$$\Rightarrow \cos\frac{A}{2} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \frac{A}{2} = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \frac{A}{2} = 30^\circ$$

$$\Rightarrow A = 60^\circ$$

92) In Young's experiment with one source and two slits, one of the slits is covered with black paper. Then

- A) The fringes will be darker    B) the fringes will be narrower    C) the fringes will be broader  
D) No fringes will be obtained and the screen will have uniform illumination

Correct Answer: D

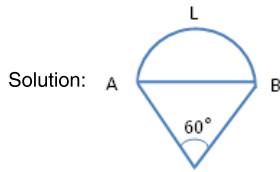
Solution: For interference, we require two coherent sources. When one of the slits is covered, the fringes disappear and there is uniform illumination on the screen.

93) A magnetic wire bent into an arc of a circle subtends  $60^\circ$  at the centre of curvature and has a magnetic moment of  $24 A - m^2$ , If it is made straight, the magnetic moment becomes

- A)  $4\pi A - m^2$     B)  $8\pi A - m^2$     C)  $24\pi A - m^2$     D)  $12\pi A - m^2$



Correct Answer: B



When the magnet is bent in the form of an arc of a circle

Length of the arc  $L = \text{Length of the magnet} = r\theta$

Angle subtended by the magnet at the centre  $\theta = 60^\circ = \frac{\pi}{3}$

$$\Rightarrow r = \frac{L}{\frac{\pi}{3}} = \frac{3L}{\pi}$$

Let pole strength of the magnet be 'm'

Magnetic moment = Magnetic length  $\times$  Pole strength

The magnetic moment of circular magnet  $M = m \times \text{distance between the poles}$ .

Distance between the poles =  $AB = 2r \sin\left(\frac{\theta}{2}\right)$

$$\Rightarrow M = m \times 2r \sin\frac{\theta}{2} = 2mr \sin 30^\circ = mr = m \times \frac{3L}{\pi} = 24$$

$$\Rightarrow mL = 8\pi$$

If the wire is made straight then its magnetic moment  $M = mL = 8\pi$

94) A charge 'q' is placed at the centre of the open end of a cylindrical vessel. The flux of electric field through the surface of vessel is

- A)  $\frac{q}{\epsilon_0}$     B)  $\frac{q}{2\epsilon_0}$     C)  $\frac{q}{3\epsilon_0}$     D)  $\frac{q}{4\epsilon_0}$

Correct Answer: B

Solution: Given that, A charge q is placed at the center of open end of a cylindrical vessel, we have to find the flux through the surface of the vessel. So, when charge q is placed at the center of open end of a cylindrical vessel then only half of the charge will contribute to the flux, because half will lie inside the surface and half will lie outside the surface.

so, flux through the surface of vessel is  $\frac{q}{2\epsilon_0}$

$$\phi = \frac{1}{2} \frac{q}{\epsilon_0}$$

95) Three charged particles are initially in position 1. They are free to move and they come in position 2 after some time. Let  $U_1$  and  $U_2$  be the electrostatic potential energies in position 1 and 2. Then

- A)  $U_1 > U_2$     B)  $U_2 > U_1$     C)  $U_1 = U_2$     D)  $U_2 \geq U_1$

Correct Answer: A

Solution: The system of particles when left to themselves tend to move towards the position where the potential energy is minimum for more stability.

$$\therefore U_2 < U_1$$

96) The emf of a cell is  $2V$  and its internal resistance is  $2\Omega$ . A resistance of  $8\Omega$  is joined to the battery in parallel. This is connected to the secondary circuit of a potentiometer. If  $1V$  standard cell balances for 100 cm of potentiometer wire, the balance point of the above cell is

- A) 120 m    B) 240 cm    C) 160 cm    D) 116 cm

Correct Answer: C

Solution: Internal resistance refers to the opposition to the flow of current offered by the cells and batteries themselves resulting in the generation of heat. Internal resistance is measured in Ohms. The relationship between internal resistance (r) and emf (E) of cell s given by.

$$E = i(r + R)$$

Where,  $E$  = EMF i.e. electromotive force ( $V$ ),  $i$  = current ( $A$ ),  $R$  = Load resistance, and  $r$  is the internal resistance of cell measured in ohms.

On rearranging the above equation we get;  $E = iR + ir$  or,  $E = V + ir$

In the above equation,  $V$  is the potential difference across the cell when the current ( $i$ ) is flowing through the circuit.

Note: The emf ( $E$ ) of a cell is always greater than the potential difference across the cell

$$i = \frac{E}{R+r} = \frac{2}{8+2} = 0.2A$$

$$V = E - ir = 2 - 0.2 \times 2 = 1.6V$$

For  $1V = K(100) \dots \dots (1)$  where  $K$  is the potential gradient

$$\Rightarrow K = \frac{1}{100}V/cm$$

$$V = V_B - V_A = Kl_2 \dots \dots (2)$$

Dividing equation (1) by (2), we get

$$\frac{1}{V} = \frac{100}{l_2}$$

$$\Rightarrow \frac{1}{1.6} = \frac{100}{l_2}$$

$$\Rightarrow l_2 = 160cm$$

97) If the displacement ( $x$ ) and velocity ( $v$ ) of a particle executing simple harmonic motion are related through the expression  $4v^2 = 25 - x^2$ , then its time period is

- A)  $\pi$     B)  $2\pi$     C)  $4\pi$     D)  $6\pi$

Correct Answer: C

Solution: Relation between displacement and velocity of a particle executing SHM is  $v = \omega\sqrt{A^2 - x^2}$

Given relation

$$4v^2 = 25 - x^2$$

$$v^2 = \frac{1}{4}(25 - x^2)$$

$$v = \frac{1}{2}\sqrt{25 - x^2}$$

comparing with  $v = \omega\sqrt{A^2 - x^2}$

$$\omega = \frac{1}{2}$$

$$\text{Time period} = T = \frac{2\pi}{\omega} = 4\pi \text{ sec}$$

98) A wire is stretched by 5 mm when it is pulled by a certain force. If the wire of same material but of double the length and double the diameter be stretched by the same force, the elongation in wire will be

- A) 2.5 mm    B) 5 mm    C) 10 mm    D) 40 mm

Correct Answer: A

Solution: Young's modulus  $Y = \frac{Fl}{Ae} = \frac{Fl}{\pi r^2 e}$

$l$  = Length of the wire,  $r$  = radius of the wire,  $d$  = diameter of the wire

$e$  = elongation of the wire.

Material is same  $\Rightarrow$  Young's modulus is the same.

Applied force  $F$  is the same,  $\pi$  is constant.

$$\Rightarrow \frac{l}{r^2 e} = k$$

$$\Rightarrow \frac{l_1}{r_1^2 e_1} = \frac{l_2}{r_2^2 e_2}$$

Given: Length and diameter are doubled.

$$l_2 = 2l_1, d_2 = 2d_1 \Rightarrow r_2 = 2r_1$$

$$\Rightarrow \frac{l}{e_1} = \frac{l}{2e_2}$$

$$\Rightarrow \frac{e_2}{e_1} = \frac{1}{2}$$

$$\Rightarrow e_2 = \frac{e_1}{2} = \frac{5}{2} = 2.5 \text{ mm}$$

99) When a capillary tube of inner radius 'r' is dipped vertically in a liquid having surface tension T and density ' $\rho$ '. The heat evolved is where angle of contact =  $0^\circ$

- A)  $\frac{2\pi T}{\rho g}$     B)  $\frac{\pi T^2}{\rho g}$     C)  $\frac{2\pi T^2}{\rho g}$     D)  $\frac{4\pi T^2}{\rho g}$

Correct Answer: C

Solution: Height of liquid level in the capillary tube  $h = \frac{2T \cos \theta}{r \rho g} = \frac{2T}{r \rho g}$

since angle of contact  $\theta = 0^\circ$

$\rho$  = density of the liquid.

r = Radius of the capillary tube

When the water rises up, work is done by surface tension.

$$W = 2\pi r T \cos \theta h = 2\pi r T h, \text{ since } \theta = 0^\circ$$

$$W = \frac{4\pi T^2}{\rho g}$$

Rise in P.E of water column  $U = mg \left(\frac{h}{2}\right)$

$$W = (\pi r^2 \rho h) g \left(\frac{h}{2}\right) = \frac{\pi r^2 \rho g h^2}{2} = \frac{2\pi T^2}{\rho g}$$

Heat evolved = work done by the surface tension - energy stored potential energy

$$= \frac{4\pi T^2}{\rho g} - \frac{2\pi T^2}{\rho g} = \frac{2\pi T^2}{\rho g}$$

100) A tank of height 20m is full of water. There is a hole of a cross-sectional area  $2 \text{ cm}^2$  in its bottom. The volume of water that will come out from this hole per second is ( $g = 10 \text{ m/sec}^2$ )

- A)  $4 \times 10^{-3} \text{ m}^3 \text{ sec}^{-1}$     B)  $2 \times 10^{-3} \text{ m}^3 \text{ sec}^{-1}$     C)  $0.5 \times 10^{-3} \text{ m}^3 \text{ sec}^{-1}$   
D)  $10^{-3} \text{ m}^3 \text{ sec}^{-1}$

Correct Answer: A

Solution: Height of the tank = h = 20m

$$\text{Velocity of flux} = v = \sqrt{2gh} = \sqrt{2 \times 10 \times 20} = 20 \text{ m/s}$$

$$\text{Area of cross section } A = 2 \text{ cm}^2 = 2 \times 10^{-4} \text{ m}^2$$

Volume of water that will come out of the hole per sec = AV

$$= 2 \times 10^{-4} \times 20 = 40 \times 10^{-4} \text{ m}^3/\text{s} = 4 \times 10^{-3} \text{ m}^3/\text{s}$$

101) The coefficient of real expansion of a liquid is found to be three times the coefficient of apparent expansion of the same liquid in a container. Then the ratio of the coefficient of linear expansion of the container to the coefficient of real expansion of the liquid is

- A) 2:9    B) 2:3    C) 1:3    D) 1:9

Correct Answer: A

Solution: Coefficient of real expansion =  $\gamma_r = 3\gamma_a$

Coefficient of apparent expansion =  $\gamma_a$

Coefficient of apparent expansion =  $\alpha$

$$\Rightarrow \gamma_r = \gamma_a + 3\alpha$$

$$\Rightarrow \gamma_a = \gamma_r - 3\alpha$$

$$\Rightarrow \gamma_r = 3\gamma_r - 9\alpha$$

$$\Rightarrow 9\alpha = 2\gamma_r$$

$$\Rightarrow \frac{\alpha}{\gamma_r} = \frac{2}{9}$$

102) An air bubble doubles in radius on rising from the bottom of a lake to its surface. Assuming that the bubble rises slowly and the atmospheric pressure to be equal to the column of water of height  $H$ , the depth of the lake is

- A)  $4H$     B)  $5H$     C)  $7H$     D)  $14H$

Correct Answer: C

Solution: When the bubble is at the surface of the water only pressure works on it is atmospheric pressure.

When the bubble is at the bottom of the water pressure works on it and atmospheric pressure.

$$P_1 V_1 = P_2 V_2$$

Let atmospheric pressure be  $P_a$

$$\text{Volume of the spherical bubble } V = \frac{4\pi r^3}{3}$$

Pressure at the bottom =  $P_a$  + Pressure due to the water.

Depth of the lake =  $h$

$$\text{Pressure due to the water} = h d g$$

Let density of the water  $d = \rho$

$$P_1 V_1 = (P_a + \rho g h) \frac{4\pi r^3}{3}$$

When the bubble reaches surface then its radius becomes 2 times the radius of the bubble.

$$P_2 V_2 = P_a \times \frac{4}{3} \pi (2r)^3$$

$$P_1 V_1 = P_2 V_2 \Rightarrow 8P_a = P_a + \rho g h$$

$$\Rightarrow 7P_a = \rho g h$$

$$p_a = \rho g H$$

$$\Rightarrow h = 7H$$

103) A tap supplies water at  $10^\circ C$  and another tap at  $100^\circ C$ . How much hot water in kilograms, must be taken so that we get  $20\text{kg}$  water at  $35^\circ C$ ?

- A) 7.2    B) 10    C) 5.6    D) 14.4

Correct Answer: C

Solution: By the principle of the method of mixtures states that the heat lost by a hot body is equal to the heat gained by the cold body when they are mixed together and attain the same temperature.

Heat lost by water due to  $100^\circ C$  = Heat gained by water at  $10^\circ C$

Let  $m_h$  be the amount of water at  $100^\circ C$ ,  $m_c$  be the mass of water at  $10^\circ C$

Let 's' be the specific heat of water .

$$m_h s (100 - 35) = m_c s (35 - 10)$$

$$\Rightarrow m_c + m_h = 20$$

$$\Rightarrow m_c = 20 - m_h$$

$$\Rightarrow m_h \times 65 = (20 - m_h) \times 25$$

$$\Rightarrow 13m_h = (20 - m_h)5$$

$$\Rightarrow 13m_h = 100 - 5m_h$$

$$\Rightarrow 18m_h = 100$$

$$\Rightarrow m_h = \frac{100}{18} = 5.55 \text{ kg}$$

104) A vessel contains  $15\text{gm}$  of a gas at a pressure  $P$  and temperature  $327^\circ C$  the gas leaks through a small hole. The mass of the gas leaked out when the pressure is  $\frac{P}{3}$  and the temperature is  $300\text{K}$  is —

- A)  $10\text{ gm}$     B)  $5\text{ gm}$     C)  $2.5\text{ gm}$     D)  $9\text{ gm}$

Correct Answer: B

Solution: Ideal gas equation  $PV = nRT$

$$\Rightarrow \frac{P_1}{n_1 T_1} = \frac{P_2}{n_2 T_2} \text{ since R is constant.}$$

$$\Rightarrow \frac{P_1}{m_1 T_1} = \frac{P_2}{m_2 T_2}$$

Initial temperature  $T_1 = 327^\circ C = 327 + 273 = 600K$

$$\Rightarrow \frac{P}{15 \times 600} = \frac{P}{3 \times 300 \times m_2}$$

$$\Rightarrow m_2 = 10gm$$

Final mass of gas  $m_2 = 10gm$

Initial mass of gas  $m_1 = 15gm$

mass of gas leaked out =  $15 - 10 = 5gm$

105) If the temperature of the sun were to increase from T to 2T and its radius from R to 2R, then the ratio of the radiant energy received on earth to what it was previously will be

- A) 4    B) 16    C) 32    D) 64

Correct Answer: D

Solution: By Stefan's law power radiated by sun =  $E = \sigma A e_\lambda T^4$

Where  $\sigma$  is Stefan's Boltzmann constant, T is temperature, A is area.

The surface area of the sphere of radius R is  $4\pi R^2$

If the radius of the sun is R. then power radiated by the sun is  $P_1 = \sigma \times 4\pi R^2 \times e_\lambda \times T^4$

If the radius of the sun is 2R. then power radiated by the sun is

$$P_2 = \sigma \times 4\pi (2R)^2 \times e_\lambda \times (2T)^4 = 64P_1$$

Energy received on the earth per unit time per unit area  $\frac{P_i}{4\pi r^2}$  where r is the distance between sun and earth

Energy received by the earth per unit time with  $a_E$  surface area  $E_1 = \frac{P_1 a_E}{4\pi r^2}$  When the sun is at temperature T

Energy received by the earth per unit time with  $a_E$  surface area  $E_2 = \frac{P_2 a_E}{4\pi r^2}$  When the sun is at temperature 2T

$$\frac{E_1}{E_2} = \frac{P_1}{P_2} = \frac{1}{64}$$

$$\Rightarrow E_2 = 64E_1$$

106) In a particular system, the units of length, mass, and time are chosen to be 10 cm, 10 gm, and 0.1sec respectively. The unit of force in this system will be

- A) 0.1 N    B) 1 N    C) 10 N    D) 100 N

Correct Answer: A

Solution:

107) The range of voltmeter of resistance  $300\Omega$  is  $5V$ . The resistance to be connected to convert it into an ammeter of range  $5A$  is

- A)  $1\Omega$  in series    B)  $1\Omega$  in parallel    C)  $0.1\Omega$  in series    D)  $0.1\Omega$  in parallel

Correct Answer: B

Solution: Since an ammeter must have very less resistance. So, a voltmeter can be converted into an ammeter by connecting a very small shunt resistance in parallel with the voltmeter resistance.

Let resistance of voltmeter be R.

$$\text{Current in the voltmeter } i_v = \frac{V}{R} = \frac{5}{300} = \frac{5}{3 \times 10^2} A$$

Let shunt resistance be S.

When shunt is connected to the voltmeter in parallel, then current in the circuit divided in the ratio

$$\frac{1}{S} : \frac{1}{R} = R : S$$

Let 'i' be the current in the ammeter = 5A

$$\text{Current in the voltmeter } i_v = \frac{iS}{R+S}$$

$$\Rightarrow \frac{5}{3 \times 10^2} = \frac{5S}{(300+S)}$$

$$300S = 300 + S \Rightarrow 299S = 300 \Rightarrow S = \frac{300}{299}$$

$$\Rightarrow S \approx 1\Omega$$

108) In a series  $LCR$  circuit  $R = 10\sqrt{3}\Omega$  and the impedance  $Z = 20\Omega$ . Then the phase difference between the current and the voltage is

- A)  $60^\circ$     B)  $30^\circ$     C)  $45^\circ$     D)  $90^\circ$

Correct Answer: B

Solution: Impedance (Z), reactance (X) and resistance (R) are connected by the relation

$$Z^2 = R^2 + X^2$$

$$\Rightarrow 400 = 300 + X^2$$

$$\Rightarrow X = 10\Omega$$

We know that  $\tan\phi = \frac{X}{R} = \frac{10}{10\sqrt{3}}$  where  $\phi$  is angle between Reactance and resistance.

$$\Rightarrow \phi = \frac{\pi}{6} = 30^\circ$$

109) If  $\lambda_0$  is the de-Broglie wavelength for a proton accelerated through a potential difference of  $100V$ . The de-Broglie wavelength for  $\alpha$ -particle accelerated through the same potential difference is

- A)  $2\sqrt{2}\lambda_0$     B)  $\frac{\lambda_0}{2}$     C)  $\frac{\lambda_0}{2\sqrt{2}}$     D)  $\frac{\lambda_0}{\sqrt{2}}$

Correct Answer: C

Solution: kinetic energy of a particle  $E = \frac{p^2}{2m}$

$$\Rightarrow p = \sqrt{2mE}$$

Here  $E = V$  = Proton is accelerated through a potential difference 100 V

$$\text{De-broglie wavelength of a particle } \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mV}}$$

$h$  is plancks constant and both proton and  $\alpha$ -particle are accelerated to the same potential. Hence  $E$  is same.

$$\Rightarrow \lambda \propto \sqrt{\frac{1}{m}}$$

$\lambda_p$  = wave length of proton

$\lambda_\alpha$  = wave length of  $\alpha$  particle.

$m_p$  = mass of proton.

$m_\alpha$  = Mass of alpha partilce.

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha}{m_p}} = \sqrt{\frac{4}{1}} = \frac{2}{1}$$

$$\lambda_\alpha = \frac{\lambda_p}{2}$$

110) The de - Broglie wavelength of a particle moving with a velocity  $225 \times 10^8 \text{ m/s}$  is equal to the wavelength of a pho ton. The ratio of kinetic energy of the particle to the energy of the photon is

- A)  $\frac{1}{8}$     B)  $\frac{3}{8}$     C)  $\frac{5}{8}$     D)  $\frac{7}{8}$

Correct Answer: B

Solution:  $\lambda = \frac{h}{p}$

Given  $\lambda_{particle} = \lambda_{photon}$

$\rho_{particle} = \rho_{photon}$

$E_{particle} = \frac{1}{2}m_{particle}V^2 = \frac{1}{2}(m_{particle}V)V = \frac{1}{2}\rho_{particle}V$

$E_{photon} = m_{photon}C^2 = (m_{photon}C)C = \rho_{photon}C$

$E_{photon} = m_{photon}C^2 = (m_{photon}C)C = \rho_{photon}C$

$$\frac{E_{particle}}{E_{photon}} = \frac{V}{2C}$$

$$= \frac{2.25 \times 10^8}{2 \times 3 \times 10^8} = \frac{2.25}{2 \times 3} = \frac{2.25}{6} = \frac{225}{600} = \frac{3}{8}$$

111) The width of for bidden gap in silicon crystal is 1.1eV. When the crystal is converted into P-type semiconductor, then the distance of fermi energy level from valance band is

- A) equal to 0.55eV    B) equal to 1.1eV    C) less than 0.55eV    D) greater than 0.55eV

Correct Answer: C

Solution:  $E_f$  = Fermi energy level for silicon,

$E_i$  = Fermi energy level for intrinsic semiconductor,

$E_g$  = energy band gap for silicon,

$E_c$  = energy of conduction band.

Now, for n-type semiconductors,  $E_f > E_i$

$$E_c - E_f = \frac{E_g}{2} - (E_f - E_i)$$

$$E_c - E_f = \frac{E_g}{2} - (E_f - E_i)$$

$$E_c - E_f = 0.55 - (E_f - E_i)$$

In intrinsic semi conductor Fermi energy level lies in the middle of the for bidden gap i.e 5.5 ev.

When the crystal is connected in the either P-type semiconductor or n-type semi conductor fermi energy level is less than 0.55 e V

112) If the height of the transmitting tower increases by 44% then the area to be covered increases by

- A) 22 %    B) 44 %    C) 66%    D) 88%

Correct Answer: B

Solution: Approximate distance covered by antenna  $d_{max} = \sqrt{2Rh}$

Where R is the radius of the earth, h is height of antenna.

2R is constant

$$\Rightarrow d_{max} \propto \sqrt{h}$$

$d_{max}$  = maximum distance covered by antennna

Area covered by the antenna  $A = \pi r^2 \Rightarrow r \propto \sqrt{A}$

$$\Rightarrow \sqrt{A} \propto \sqrt{h}$$

$$\Rightarrow A \propto h$$

Thus, if the height of the tower is increased by 44%, then the area covered is increased by 44%

113) If  ${}_{92}U^{238}$  changes to  ${}_{85}At^{210}$  by a series of  $\alpha$  and  $\beta$  decays, the number of  $\alpha$  and  $\beta$  decays undergone is

- A) 7 & 5    B) 7 & 7    C) 5 & 7    D) 7 & 9

Correct Answer: B

Solution: Alpha decay or  $\alpha$ -decay is a type of radioactive decay in which an atomic nucleus emits an alpha particle i.e helium nucleus and thereby transforms or 'decays' into a different atomic nucleus, with a mass number that is reduced by four and an atomic number that is reduced by two.

The mass number of Uranium = 238

The mass number of Astatine = 210

An alpha particle is a doubly charged Helium ion.  $\alpha = {}_2\text{He}^4$

Mass number of  $\alpha = 4$

Atomic number of  $\alpha = 2$

$\beta$  particle

Mass number of  $\beta = 0$

Change in atomic number by  $\beta = \pm 1$

Change in mass number because of  $\alpha$  particles.

$$\text{Number of } \alpha\text{-particles} = \frac{238-210}{4} = \frac{28}{4} = 7$$

$$\text{Number of } \beta\text{-particles} = \frac{85-78}{1} = 7$$

$$\text{Total number of protons left } 92 - 2 \times 7 = 92 - 14 = 78$$

But total number of protons left over = 85

$$\text{Total number of } \beta \text{ decay is } 85 - 78 = 7$$

114) The potential barrier in the depletion layer is due to

- A) ions    B) holes    C) electrons    D) both 2 and 3

Correct Answer: A

Solution: After joining p-type and n-type semiconductors, electrons from the n-region near the p-n interface tend to diffuse into the p-region.

As electrons diffuse, they leave positively charged ions (donors) in the n-region.

Likewise, holes from the p-type region near the p-n interface begin to diffuse into the n-type region, leaving fixed ions (acceptors) with negative charge.

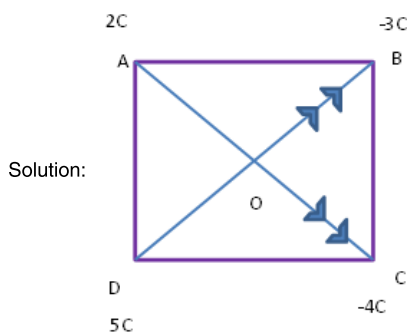
Hence, the region (depletion) nearby the p-n interfaces lose their neutrality and become charged due to ions.

Due to diffusion of holes and electrons across the junction, the voltage is developed which is known as potential barrier

115) Four point charges  $2C$ ,  $-3C$ ,  $-4C$  &  $5C$  respectively are placed at the corner of a square. If  $\vec{E}$  &  $V$  represent the electric field strength & potential at the centre of square then

- A)  $\vec{E} = 0$ ,  $V = 0$     B)  $|\vec{E}| = 0$   $V \neq 0$     C)  $|\vec{E}| \neq 0$   $V = 0$     D)  $|\vec{E}| \neq 0$   $V \neq 0$

Correct Answer: C



Solution:

$\vec{E}_{net}$  is a vector sum of fields



Let 'a' be the side of a square.

$$AC = \sqrt{2}a \Rightarrow AO = \frac{a}{\sqrt{2}} = x$$

The electric field at O due to A towards OC and due to C also towards OC

$$\text{Electric field due to charges present at A and C } \vec{E} = \frac{K(2C)}{x^2} + \frac{K(-4C)}{x^2} = \frac{K(-2C)}{x^2} = E_1$$

The electric field at O due to D towards OB and due to B also towards OB

$$k = \frac{1}{4\pi\epsilon_0}$$

$$\text{Electric field due to charges present at D and B } \vec{E} = \frac{K(5C)}{x^2} + \frac{K(-3C)}{x^2} = \frac{K(2C)}{x^2} = E_2$$

Net electric field  $E = \sqrt{E_1^2 + E_2^2}$  since they are perpendicular

Since diagonals are perpendicular.

$E_1, E_2$  are magnitudes hence net electric field is not zero.

Potential  $V = \frac{q}{4\pi\epsilon_0 r}$  is a scalar

$$\text{Net potential} = V = \frac{1}{4\pi\epsilon_0} \frac{1}{r} [2 - 3 - 4 + 5] = 0$$

116) A transverse wave is described by the equation  $y = y_o \sin 2\pi(ft - \frac{x}{\lambda})$ . The maximum particle velocity is equal to four times the wave velocity if

- A)  $\lambda = \frac{\pi y_o}{4}$     B)  $\lambda = \frac{\pi y_o}{2}$     C)  $\lambda = \pi y_o$     D)  $\lambda = 2\pi y_o$

Correct Answer: B

Solution: Equation of the wave  $y = y_o \sin 2\pi(ft - \frac{x}{\lambda})$

$$y = A \sin 2\pi(\frac{t}{T} - \frac{x}{\lambda})$$

$$\Rightarrow V_{particles} = W \sqrt{A^2 - x^2}$$

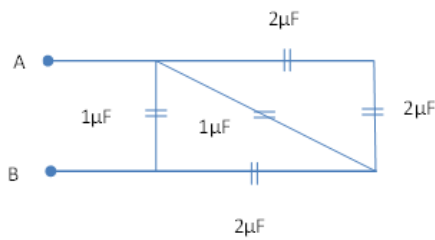
Maximum particle velocity =  $V_{max} = WA$  i. e at  $x = 0$

Wave velocity  $V = n\lambda$

The maximum particle velocity is equal to four times the wave velocity

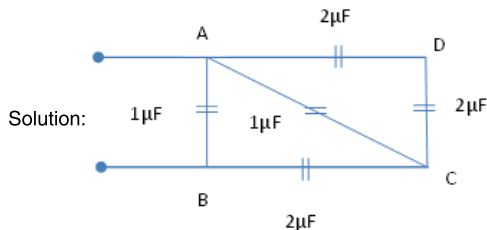
$$WA = 4n\lambda \Rightarrow \lambda = \frac{2\pi y_o}{4n} = \frac{\pi y_o}{2}$$

117) The total capacity of the system in the figure between A and B is



- A)  $1\mu F$     B)  $2\mu F$     C)  $3\mu F$     D)  $4\mu F$

Correct Answer: B



If two capacitors  $C_1, C_2$  are connected in series then effective capacitance  $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

If two equal capacitors of capacitance C are connected in series then effective capacitance  $\frac{C}{2}$

If two capacitors  $C_1, C_2$  are connected in parallel then effective capacitance  $C = C_1 + C_2$

Using reduction method.

Two capacitors  $2\mu F, 2\mu F$  between A and C are in series, hence effective capacitance is  $1\mu F$

$1\mu F$  and one more capacitor of capacitance between A and C are in parallel.

Hence their effective capacitance  $1 + 1 = 2\mu F$

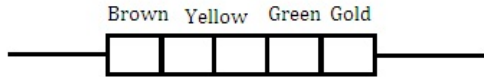
$2\mu F, 2\mu F$  are in series between A and B

Their effective capacitance =  $1\mu F$

$1\mu F$  parallel  $1\mu F$  are connected in parallel between A and B

Effective capacitance =  $2\mu F$

118) Suppose the colors on the resistor as shown in the figure are brown, yellow, green, and gold as read from left to right. Find the resistance of the resistor.



- A)  $(1.4 \pm 0.07) M\Omega$     B)  $(1.4 \pm 0.07) K\Omega$     C)  $(1.5 \pm 0.05) M\Omega$     D)  $(1.4 \pm 0.05) G\Omega$

Correct Answer: A

Solution: To remember code of colored resistor we use

BB ROY of GREAT Britain Very Good Wife

Code starts from 0

Codes of brown = 1, yellow = 4, Green = 5

Gold tolerance band of tolerance = 5

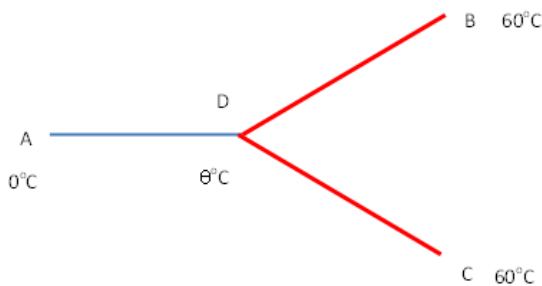
$$\Rightarrow R = 14 \times 10^5 \pm 5\% \Omega$$

$$= 1.4 \times 10^6 \pm 5\%$$

$$= 1.4 \times 10^6 \pm \frac{5}{100} \times 1.4 \times 10^6 \Omega = 1.4 \pm 0.07 \times 10^6 \Omega$$

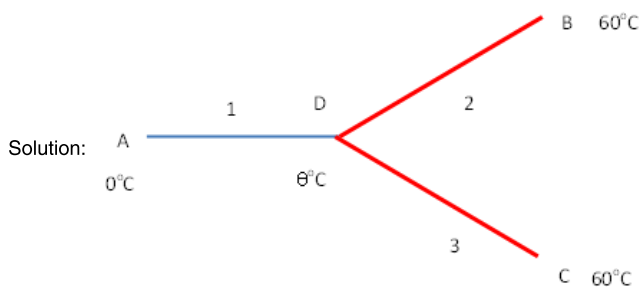
$$= 1.4 \pm 0.07 M\Omega$$

119) Three rods made of the same material and the same length and cross-sectional areas are joined as shown in the figure. The temperature of the junction D of three rods is



- A)  $40^\circ C$     B)  $50^\circ C$     C)  $60^\circ C$     D)  $80^\circ C$

Correct Answer: A



Thermal current is governed by temperature difference and thermal resistance.

The thermal resistance of a thermal conductor of length  $L$  and cross-sectional area  $A$  and thermal conductivity  $K$  is given by  $R = \frac{L}{KA}$ .

It is evident that the right rods 2 and 3 are at a higher temperature than that of the left end of the first rod.

So the heat will flow from the 2 and 3 rods to the first rod.

The Sum of the rate flow of heat from the second and third rods will be equal to the rate flow of heat in the third rod.

Let the junction temperature be  $\theta$

Let  $\theta^\circ C$  be the temperature at D. Let  $Q$  be the heat flowing per second from D to B on account of a temperature difference by conductivity.

$$\therefore Q = \frac{KA(60-\theta)}{\ell} \dots \dots (i)$$

Where  $K$ = thermal conductivity of the rod,  $A$ = Area of a cross-section of the rod,  $\ell$ = length of the rod. By symmetry, the same will be the case for heat flow from C to D.

$$\frac{Q_1}{t} + \frac{Q_2}{t} = \frac{Q}{t}$$

$$\Rightarrow 60 - \theta + 60 - \theta = \theta$$

$$\Rightarrow 120 = 3\theta \Rightarrow \theta = 40^\circ$$

$$\therefore \theta = 40^\circ$$

120) A parallel plate condenser has conducting plates of radius 12cm separated by a distance of 5mm. It is charged with a constant charging current of 0.16 A, the rate at which the potential difference between the plates change is

- A)  $1 \times 10^9 \text{ Vs}^{-1}$     B)  $2 \times 10^{10} \text{ Vs}^{-1}$     C)  $3 \times 10^{12} \text{ Vs}^{-1}$     D)  $2 \times 10^9 \text{ Vs}^{-1}$

Correct Answer: D

Solution:  $I_d = C \frac{dv}{dt}$  In steady condition  $I_d = I_c$

$$\frac{dv}{dt} = \frac{I}{C}$$

$$\text{Where } C = \frac{\epsilon_0 A}{d}$$

$$= 8.85 \times 10^{-12} \times \frac{3.142 \times 144 \times 10^{-4}}{5 \times 10^{-3}}$$

$$= 80 \text{ PF}$$

$$\frac{dv}{dt} = \frac{0.16}{80 \times 10^{-12}} = 0.002 \times 10^{12} = 2 \times 10^9 \text{ v/s}$$

121) The atomic radius of elements of which of the following series would be nearly the same

- A) Na, K, Rb, Cs    B) Li, Be, B, C    C) Fe, Co, Ni, Cu    D) F, Cl, Br, I

Correct Answer: C

Solution: Atomic radius is the distance from the center of the nucleus to the outermost shell containing electrons. In other words, it is the distance from the center of the nucleus to the point up to which the density of the electron cloud is maximum.

As we move from left to right in a period, the nuclear charge increases by 1 unit in each succeeding element while the number of shells remains the same. This enhanced nuclear charge pulls the electrons of all the shells closer to the nucleus. This makes each individual shell smaller and smaller. This results in a decrease in the atomic radius as we move from left to right in a period.

Li, Be, B, C are elements of the second period, hence the atomic radius decreases as we move from left to right.

The atomic radii of elements increase with an increase in the atomic number from top to bottom in a group. As we move down the group, the principal quantum number increases. A new energy shell is added at each succeeding element. The valence electrons lie farther and farther away from the nucleus. As a result, the attraction of the nucleus for the electron decreases. Hence, the atomic radius increases.

Na, K, Rb, Cs are elements of first group. Hence atomic radius increases down the group.

F, Cl, Br, I are also elements of the same group hence atomic radius increases down the group.

Fe, Co, Ni, Cu due to shielding of d-electrons, the effect of increased nuclear charge due to an increase in atomic no. neutralised. Consequently, the atomic radius remains almost unchanged after chromium.

122) An example for  $AB_3E_2$  representation is

- A)  $SO_3$     B)  $XeO_3$     C)  $ClF_3$     D)  $BrF_5$

Correct Answer: C

Solution: A = central atom

B = Binded atom

E = Number of Lone pairs

V = Valency of the central atom = 7

M = Number of monovalent atoms binded = 3

$$\text{Total } e^- \text{ pairs} = \frac{V+M}{2}$$

$$\text{Number of lone pair } e^- \text{ s} = \text{Total electro pairs} - \text{all surrounding atoms} = \frac{7+3}{2} = \frac{10}{2} = 5; 5 - 3 = 2.$$

123) Non polar molecule among the following compounds is

- A)  $CCl_4$     B)  $SF_4$     C)  $H_2S$     D)  $NCl_3$

Correct Answer: A

Solution:  $CCl_4$  is a non-polar molecule as the individual C-Cl bonds are polar but due to the tetrahedral shape of  $CCl_4$ , the bond polarity cancels out and thus the net dipole moment is zero.

124) If the wave length of electron and its velocity is numerically same then

- A)  $\lambda = \frac{h}{m}$     B)  $\lambda = \sqrt{\frac{h}{m}}$     C)  $\lambda = \sqrt{\frac{h}{p}}$     D)  $\lambda = \frac{h}{m^2}$

Correct Answer: B

Solution: We know, de Broglie wave length  $\lambda = \frac{h}{mv}$

But, since the wave length in one second, we have  $\lambda = v$

$$\text{Thus } \lambda = \frac{h}{m \times \lambda}$$

$$\lambda^2 = \frac{h}{m}$$

$$\Rightarrow \lambda = \sqrt{\frac{h}{m}}$$

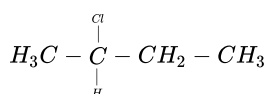
125) Which of the following is optically active?

- A) 2-chloro butane    B) 1- chloro butane    C) 1- chloro propane    D) tert. butyl chloride

Correct Answer: A

Solution: For an optical activity, the compound should have a chiral carbon. Chiral carbon is a carbon whose all four valencies are satisfied by 4 different groups.

2-chloro butane contains one chiral carbon. So, it is optically active.molecule with a chiral center



126) The compound with a lower boiling point is

- A) n - pentane    B) neopentane    C) Isopentane    D) n-hexane

Correct Answer: B

Solution: n-Pentane have a higher boiling point than neopentane. The shape of n-pentane, a straight chain, fits well together with other n-pentane molecules, forming tight, well-compacted layers, which are harder to separate.

In neopentane, which is 2,2-dimethylpropane, trying to fit neopentane molecules together is difficult because of the substituents. Because of this uneven "packing", it becomes easier to separate and the boiling point is lower.

The boiling point of neopentane is only 9.5 °C, significantly lower than those of isopentane (27.7 °C). Therefore, neopentane is a gas at room temperature and atmospheric pressure, while the other is liquid.

A straight-chain alkane has a higher boiling point than the branched one.

In straight chain alkane, the surface area of contact is more and hence the Vander Waal forces are stronger.

Also, more are the no. of carbon atoms in the straight-chain, more is the surface area of the molecule and higher is the boiling point.

Therefore n-hexane has the highest boiling point.

127) Which of the following has acidic hydrogen and liberates  $H_2$  gas with sodium

A)  $CH_3HC = CH_2$     B)  $CH \equiv CH$     C)  $CH_3 - C \equiv C - CH_3$     D)  $CH_2 = CH_2$

Correct Answer: B

Solution: Hydrogen atoms in ethyne  $HC \equiv CH$  are attached to the sp hybridized carbon atoms which are more electronegative. Thus, a hydrogen atom can be liberated as proton more easily and ethyne behaves as an acid, thus easily reacts with base  $NaNH_2$  and forms sodium acetylide.

Alkynes are called unsaturated compounds because the carbon atoms that they contain are bonded to fewest hydrogen atoms than they can possibly hold.

128) The volume of  $H_2$  at STP required to convert 14 gm of  $N_2$  completely into  $NH_3$  is

A) 11.2 lit    B) 22.4 lit    C) 33.6 lit    D) 44.8 lit

Correct Answer: C

Solution:  $N_2 + 3H_2 \rightleftharpoons 2NH_3$

At STP one mole of gas takes up 22.4liters

Molecular weight of  $N_2 = 2 \times 14 = 28$

1mole of nitrogen requires 3moles of hydrogen  $H_2$

i.e 28gm of  $N_2$  requires  $3 \times 22.4$  liters of  $H_2$

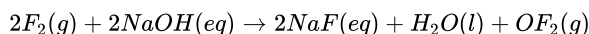
i.e 14gm of  $N_2$  requires  $3 \times 22.4 \times \frac{14}{28} = 33.6$ liters

129) Fluorine is passed into cold dilute  $NaOH$  solution. What are the oxidation numbers of fluorine in the products formed ?

A) + 1, + 3    B) + 1, + 5    C) -1, + 3    D) -1, -1

Correct Answer: D

Solution: Fluorine reacts with cold sodium hydroxide solution to produce oxygen difluoride as shown in the following chemical equation



Fluorine exhibits always '-1' oxidation state.

130) The following relationship is correct between critical pressure ( $P_c$ ), critical temperature, ( $T_c$ ) and critical volume ( $V_c$ )

- A)  $P_c V_c = \frac{3}{2} RT_c$     B)  $P_c V_c = RT_c$     C)  $P_c V_c = \frac{3}{8} RT_c$     D)  $P_c V_c = \frac{8}{3} RT_c$

Correct Answer: C

Solution: Critical temperature =  $T_c = \frac{8a}{27Rb}$

Critical pressure  $P_c = \frac{a}{27b^2}$

Critical volume  $V_c = 3b$

$P_c V_c = \frac{a}{27b^2} \times 3b = \frac{a}{9b}$

$RT_c = R \times \frac{8a}{27Rb} = \frac{8}{27b}$

$\frac{3}{8} RT_c = \frac{a}{9b}$

131) Which of the following artificial sweetener is a chloro derivative of sucrose

- A) Alitame    B) Aspartame    C) Saccharine    D) Sucralose

Correct Answer: D

Solution: Sucralose is an artificial sweetener that contains chlorine. It is a trichloro derivative of sucrose. It is 600 times sweeter than cane sugar.

132) The hybridisation of chlorine in  $HClO_3$  molecule is

- A)  $sp$     B)  $sp^2$     C)  $sp^3$     D)  $sp^3d$

Correct Answer: C

Solution: The hybridization number of a molecule can be calculated according to the formula -

$H = GA + \frac{VE - V - C}{2}$ , where H denotes the hybridization number, GA denotes the group atoms attached to the central atom, VE denotes the valence electrons of the central atom, V denotes the valency of the central atom and C denotes the charge over the molecule.

The central atom in  $ClO_3^-$  is chlorine. It is attached to three oxygen atoms so the number of group atoms attached is 3. The valence electrons over the chlorine atom is 7 and the chlorine atom is attached to three oxygen atoms by means of three double covalent bonds so its valency is 6. Also, there is a negative charge present over the molecule, so its charge is -1. Putting the values we get

$$H = 3 + \frac{7 - 6 - (-1)}{2} = 3 + \frac{1 + 1}{2} = 3 + \frac{2}{2} = 3 + 1 = 4$$

So the molecule is having a hybridization number of 4 and thus its hybridization state is  $sp^3$  that is the molecule of  $ClO_3^-$  is  $sp^3$  hybridized.

133) Assertion(A): Nitrogen cannot form  $NCl_5$  but phosphorus gives  $PCl_5$

Reason(R): Nitrogen has no vacant 'd' orbitals but phosphorus has vacant d orbitals

- A) A, R are true and R is a correct explanation of A  
 B) A, R are true but R is not correct explanation of A    C) A is true, R is false  
 D) A is false, R is true

Correct Answer: A

Solution: Nitrogen cannot form  $NCl_5$  but phosphorus gives  $PCl_5$  since Nitrogen has no vacant 'd' orbitals but phosphorus has vacant d orbitals.

134) The hydrogen electrode is dipped in a solution of  $p^H = 3$  at  $25^\circ C$ . The potential of the cell would be

- A) 0.277 V    B) 0.087 V    C) -0.177 V    D) 0.059 V

Correct Answer: C

Solution:  $p^H = 3$

$$[H_3O^+] = 10^{-pH} = 10^{-3} M$$

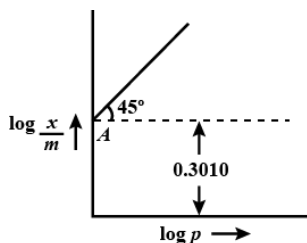
$$E_{cel} = E_{cel}^o - \frac{0.059}{n} \log \frac{P_{H_2}}{[H^+]^2}$$

$$E_{cel} = 0 - \frac{0.059}{2} \log \frac{P_{H_2}}{[10^{-3}]^2}$$

$$E_{cel} = -0.177V$$

$$\text{or } E_{cel} = -0.059p^H = -0.177V$$

135) Graph between  $\log(x/m)$  and  $\log p$  is a straight line at angle  $45^\circ$  with intercept OA as shown in figure. Then  $(x/m)$  at a pressure of 2 atm is



- A) 2    B) 4    C) 8    D) 1

Correct Answer: B

Solution: Freundlich gave an empirical relationship between the quantity of gas adsorbed by a unit mass of solid adsorbent and pressure at a particular temperature. The relationship can be expressed by the following equation:  $\Rightarrow \frac{x}{m} = k \times (p)^{\frac{1}{n}} \dots (1)$

where  $x$  is the mass of the gas adsorbed on mass  $m$  of the adsorbent at pressure  $P$ ,  $k$  and  $n$  are constants which depend on the nature of the adsorbent and the gas at a particular temperature.

Here, the value of  $n$  should be always greater than one so as to equation be valid for a range of values.

Taking logarithms for an equation (1) we get

$$\text{Equation changes to } \log\left(\frac{x}{m}\right) = \frac{1}{n} \log p + \log k$$

Equation of a line in slope form  $y = mx + c$

$\log\left(\frac{x}{m}\right)$  is taken on y-axis and  $\log p$  is taken on x-axis

From the diagram

$$\log\left(\frac{x}{m}\right) = \tan(45^\circ) \log p + \log 2$$

$$\Rightarrow \log\left(\frac{x}{m}\right) = \log p + \log 2 = \log 2p$$

$$\Rightarrow \left(\frac{x}{m}\right) = 2p$$

Hence at pressure  $p = 2 \text{ atm}$

$$\text{Value of } \frac{x}{m} = 2p = 4 \text{ atm}$$

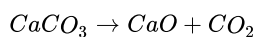
136) In the dissociation of  $CaCO_3$  in a closed vessel, the forward reaction is favoured by

- A) adding some more  $CaCO_3$     B) adding  $CaO$     C) increasing the pressure  
D) removing  $CO_2$

Correct Answer: D

Solution: According to Lechatelier's principle removing of product always favours forward reaction.

At 1200K, calcium carbonate decomposes to give carbon dioxide and calcium oxide.



137) The most unsymmetric crystal system is

A) Hexagonal    B) Triclinic    C) Monoclinic    D) Orthorhombic

Correct Answer: B

Solution: Seven crystal systems: On the basis of edge length and interaxial angle, crystal system is of seven types- cubic, tetragonal, orthorhombic, monoclinic, hexagonal, triclinic, rhombohedral.

For triclinic crystal system

$a, b, c$  are edges and  $\alpha, \beta, \gamma$  are bond angles.

$a \neq b \neq c, \alpha \neq \beta \neq \gamma$

The most unsymmetrical crystal system is triclinic while the most symmetrical crystal system is cubic.

138) Which of the following is a nonreducing sugar?

A) Glucose    B) Sucrose    C) Fructose    D) Maltose

Correct Answer: B

Solution: Non-reducing sugars are those which cannot donate electrons to other molecules and they cannot be used as reducing agents. Here the answer is Sucrose because it is a non-reducing sugar because the anomeric carbon of both the monosaccharide units is involved in glycoside or acetal formation.

139) In the complex  $K_2[Ni(CN)_4]$ , oxidation state of nickel is

A) +2    B) +4    C) +1    D) 0

Correct Answer: A

Solution: In the complex  $K_2[Ni(CN)_4]$ ,

Let oxidation number of nickel be 'x'

$$\Rightarrow 2 \times (+1) + x + 4 \times (-1) = 0$$

$$\Rightarrow +2 + x - 4 = 0$$

$$\Rightarrow x - 2 = 0$$

$$\Rightarrow x = +2$$

140) Molar conductance of  $Al_2(SO_4)_3$  is  $x \text{ ohm}^{-1} \text{ cm}^2 \text{ mol}^{-1}$ . Its equivalent conductance is

A)  $\frac{x}{6}$     B)  $6x$     C)  $\frac{x}{3}$     D)  $3x$

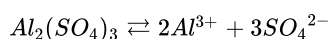
Correct Answer: A

Solution: Let the number of cations =  $n^+$

Charge of cations =  $Z^+$

$$\text{conductance } \lambda_{eq} = \left( \frac{1}{(n^+) \times (Z^+)} \right) \lambda_m$$

Given  $\lambda_m = x$



$$Z^+ = 3, n^+ = 2$$

$$\Rightarrow \lambda_{eq} = \frac{x}{2 \times 3}$$

$$\lambda_{eq} = \frac{x}{6}$$

Molar conductance =  $6 \times$  Equivalent conductance.

141) The compound which can exchange more hydrogens with  $D_2O$  is



- A)  $H_3PO_3$     B)  $H_3PO_2$     C)  $CH_3COOH$     D)  $H_3PO_4$

Correct Answer: D

Solution:  $D_2O$  can exchange proton from  $-OH$  group of given options.

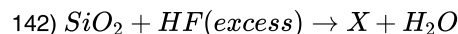
*Compound    Basicity*

$H_3PO_4$     3

$CH_3COOH$     1

$H_3PO_2$     1

$H_3PO_3$     2



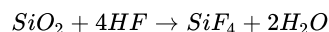
The product 'X' and covalency of the central atom in X are respectively

- A) Orthosilicic acid, 4    B) Silicon tetrafluoride, 6    C) Hydrofluorosilicic acid, 6  
D) Pyrosilicic acid, 6

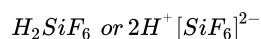
Correct Answer: C

Solution: When silicon dioxide  $SiO_2$  is heated with hydrogen fluoride HF, it forms silicon tetrafluoride  $SiF_4$ . The  $SiF_4$  formed in this reaction can further react with HF to form Hydrofluorosilicic acid.

Usually the Si-O bond is a strong bond and it resists any attack by halogen and most acids, even at a high temperature, However it is attacked by HF.



The  $SiF_4$  formed in this reaction can further react with HF to form Hydrofluorosilicic acid.



143) The most abundant inert gas in atmosphere and most reactive inert gas are respectively

- A) Ar, He    B) Ar, Xe    C) Xe, Ar    D) Xe, He

Correct Answer: B

Solution: Argon is the most abundant noble gas in the atmosphere. Xenon is more reactive among the rare gases.

144) A salt 'X' gives a white precipitate with  $NaOH$  & the precipitate dissolves in excess  $NaOH$ . The salt 'X' is

- A)  $FeSO_4$     B)  $FeCl_3$     C)  $ZnCl_2$     D)  $MnSO_4$

Correct Answer: C

Solution: Zinc chloride gives a white precipitate of  $Zn(OH)_2$  with sodium hydroxide which is soluble in excess of ammonia.

145) The biodegradable polymer used in making capsules is

- A) PHBV    B) Poly glycolic acid    C) Dextran    D) Silicone elastomer

Correct Answer: A

Solution: PHBV is Poly  $\beta$  hydroxybutyrate - co -  $\beta$ -hydroxy valerate. PHBV is used in specialty packaging, orthopedic devices, and in controlled release of drugs. PHBV undergoes bacterial degradation in the environment.

146) In the reaction  $Na_2S_2O_3 + Cl_2 + H_2O \rightarrow Na_2SO_4 + S + 2HCl$ , the equivalent weight of hypo is (molecular wt = 248)

- A) 158    B) 79    C) 248    D) 124

Correct Answer: D

Solution:  $Na_2S_2O_3 + H_2O + Cl_2 \rightarrow Na_2SO_4 + 2HCl + S$

$Na_2S_2O_3$  acts as a reductant to form  $Na_2SO_4$

Oxidation state of S in  $Na_2S_2O_3$  is +2

Oxidation state of S in  $Na_2S_2O_4$  is +6

∴ Change in oxidation state of for each sulphur S = 2

Equivalent weight =  $\frac{\text{Molecular weight}}{2} = \frac{248}{2} = 124$

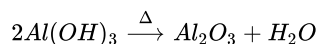
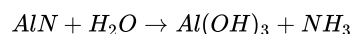
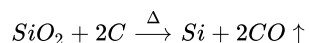
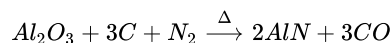
147) White bauxite is purified by

- A) Bayer's process    B) Hall's process    C) Serpeck's process    D) Hoop's process

Correct Answer: C

Solution: Serpeck's process is used for the bauxite ore which contains silica impurity. The powdered bauxite ore is mixed with coke and heated at high temperature in the presence of nitrogen.

White bauxite containing the impurity of silica only is removed by heating with coke in the atmosphere of nitrogen.



148) The work done on the surroundings is 9 joules. 45J heat is supplied to the system. The change in internal energy is

- A) -36J    B) -54J    C) +36J    D) +54 J

Correct Answer: C

Solution: According to first law of thermodynamics  $\Delta E = \Delta Q + \Delta W$

Where  $\Delta E$  is the change in internal energy of the system

$\Delta Q$  = is the heat involved during the process

$\Delta W$  = work done during the process is negative

$$\Delta Q = \Delta E - 9$$

$$45 = \Delta E - 9$$

$$\Rightarrow \Delta E = 45 - 9 = 36J$$

149) The mole fraction of urea in 2 molal aqueous solutions is

- A) 0.0348    B) 0.9652    C) 0.2000    D) 0.8000

Correct Answer: A

Solution: Mole fraction X is the ratio of moles of one substance in a mixture to the total number of moles of all substances.

$$\text{Molality of the solution} = \frac{\text{Number of moles of solute}}{\text{Mass of solvent (in Kg)}}$$

Solute = urea, Solvent = water

$$\text{Molecular weight of water } H_2O = 2 + 16 = 18$$

Let 'x' be the mole fraction of the solute, then mole fraction of the solvent =  $1 - x$

Mass of the solvent = Number of moles of the water  $\times$  Molar mass of the water =  $(1 - x) \times 18g$

Molality of the solution =  $\frac{x}{(1-x) \times \frac{18}{1000}} = 2$

$$\frac{1000x}{(1-x)18} = 2$$

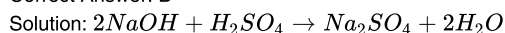
$$\Rightarrow 250x = 9(1 - x)$$

$$\Rightarrow x = \frac{9}{259} = 0.0348$$

150) Which of the following solutions can exactly neutralize 2 gms. of NaOH?

- A) 25ml of 0.1 N acid    B) 250ml of 0.2N acid    C) 100ml of 2.5 N acid    D) all the above

Correct Answer: B



Molecular weight of  $NaOH = 23 + 16 + 1 = 40$

Molecular weight of  $H_2SO_4 = 2 + 32 + 64 = 98$

So 80 gm. of NaOH will react with = 98 gm of  $H_2SO_4$

Molarity =  $0.2 = \frac{\text{mass of the substance}}{\text{molecular weight} \times \text{volume in liters}}$

$$0.2 = \frac{x}{40} \times \frac{25}{1000} \Rightarrow x = 0.2gm$$

80 gm NaOH will react with = 98 gm  $H_2SO_4$

0.2gm of NaOH react with

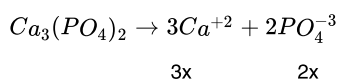
$$= \frac{0.2}{80} \times 98 = 0.245gms \text{ of } H_2SO_4$$

151) The solubility of calcium phosphate in water is ' $x'$  mol  $L^{-1}$  at  $25^\circ C$ . Its solubility product is equal to

- A)  $108x^2$     B)  $36x^3$     C)  $36x^5$     D)  $108x^5$

Correct Answer: D

Solution: Let solubility be ' $x$ '



Solubility product,  $K_{sp} = (3x)^3(2x)^2 = 108x^5$

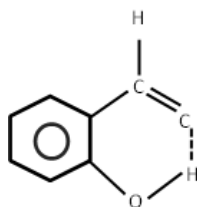
152)  $C_6H_5 - OH + CHCl_3 + 3NaOH \xrightarrow{65^\circ C} X + 3NaCl + 2H_2O$  The compound 'X'

- A) has intramolecular hydrogen bond    B) can be purified by steam distillation    C) is salicylaldehyde  
D) all the above

Correct Answer: D

Solution: When phenols i.e.  $C_6H_5OH$  is treated with  $CHCl_3$  (chloroform) in the presence of NaOH (sodium hydroxide), an aldehyde group (-CHO) is introduced at the ortho position of benzene ring leading to the formation of o-hydroxybenzaldehyde. The reaction is popularly known as Reimer Tiemann's reaction.

Ortho hydroxybenzaldehyde also known as salicylaldehyde.

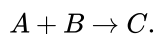
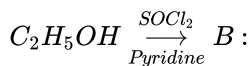
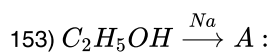


Salicylaldehyde has intramolecular hydrogen bonding.

It also can be purified by steam distillation.

Hence all options are correct

D is the correct answer.

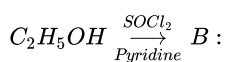
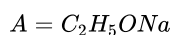
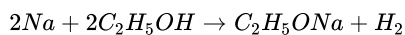


The final product 'C' forms an oxonium salt with strong mineral acids at low temperatures. The valency of Oxygen in the salt is

- A) 1    B) 2    C) 3    D) 4

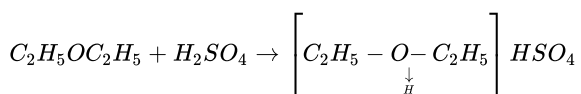
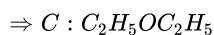
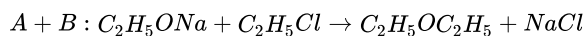
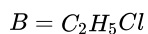
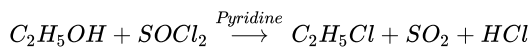
Correct Answer: C

Solution: Sodium reacts with ethanol to produce sodium ethoxide and hydrogen. The reaction proceeds at room temperature.



By this reaction pure, alkyl halide forms as a product, because other formed products are gaseous.

The reaction is



154) Which of the following has zwitter ion structure

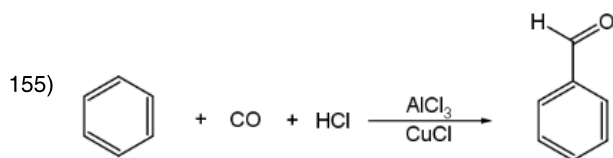
- A) p - cresol    B) salicylic acid    C) picric acid    D) Sulphanilic acid

Correct Answer: D

Solution: A zwitterion is a molecule that contains an equal number of positively-charged functional groups and negatively-charged functional groups.

Strongly acidic  $HO_3S$  group transfers proton to the weakly basic  $-NH_2$  group to form a dipolar ion (or zwitterion).

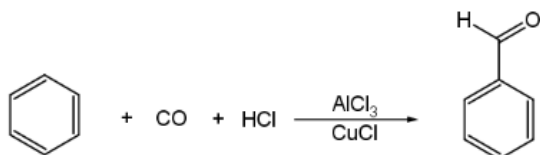




- A) Cannizzaro's reaction      B) Cross aldol condensation      C) Gattermann – Koch reaction  
D) Wacker's process

Correct Answer: C

Solution: Gattermann Koch Reaction Mechanism begins with the formation of the reactive species with the help of the acid. The overall aim of the reaction is to attach a formyl group (-CHO group) to an aromatic system. An example of the Gattermann – Koch reaction is given below.



156) What percentage of acetic acid is used for cooking purposes & known as vinegar

- A) 6 to 10%    B) 20%    C) 98%    D) 1%

Correct Answer: A

Solution: Vinegar is an aqueous solution of acetic acid .It typically contains 5–8% acetic acid by volume

157) A 0.1 g/cc Organic solution is placed in 2 decimeters long polarimeter tube showed an observed rotation  $+6^{\circ}$ . Specific rotation will be

- A)  $+3^{\circ}$     B)  $+30^{\circ}$     C)  $+0.3^{\circ}$     D)  $+60^{\circ}$

Correct Answer: B

Solution: Observed rotation  $\mu = 6^{\circ}$

Path length in decimeters and concentration must be in g/milliliter.

1cc = milliliter

Path length  $l = 2dm$

Concentration of the solution  $c = 0.1gm/cc$

$$\text{Specific Rotation} = \frac{\mu_{obs}}{c \times l} = \frac{6}{0.1 \times 2} = \frac{6}{0.2} = 30^{\circ}$$

158) Rectified spirit is a mixture of

- A) 50% ethyl alcohol and 50% water      B) 98% Ethyl alcohol and 2 % water  
C) 95.6 % ethyl alcohol and 4.4% water      D) 95.6 % water and 4.4% ethyl alcohol

Correct Answer: C

Solution: The individual substances in a mixture can be separated using different methods, depending on the type of mixture. These methods include filtration, evaporation, distillation, and chromatography.

Fractional distillation is a method for separating a liquid from a mixture of two or more liquids. For example, liquid ethanol can be separated from a mixture of ethanol and water by fractional distillation. This method works because the liquids in the mixture have different boiling points. When the mixture is heated, one liquid evaporates before the other.

A water and ethanol mixture is heated in a flask using an electric heater. Vapor forms in the air above the mixture in the flask.

The liquid collected by condensing the vapor from the top of the fractionating column cannot be pure ethanol. The best you can produce by simple fractional distillation is 95.6% ethanol.

This is due to the fact that ethanol forms azeotropic solutions with water at a concentration of 95.6% by volume. As soon as this composition is achieved, the vapor and the solution exist with same composition and no further separation occurs.

159) For an ionic crystal of general formula  $AX$  and co-ordination number 6, the value of radius ratio will be

- A) Greater than 0.73    B) In between 0.732 and 0.414    C) In between 0.41 and 0.22  
D) Less than 0.22

Correct Answer: B  
Solution:

160) An explosive compound (A) reacts with water to produce  $NH_4OH$  and  $HOCl$ . Then, the compound (A), is

- A)  $TNG$     B)  $NCl_3$     C)  $PCl_3$     D)  $HNO_3$

Correct Answer: B  
Solution:  $NCl_3 + 4H_2O \rightarrow NH_4OH + 3HOCl$   
Hence explosive product Nitrogen trichloride  $NCl_3$